Note Title 6/28/2017

A few more factoring problems:
(6.5 cont'd)

(+) same signs
want a sum of $13x^2$ for middle term

Example: $x^4 - 13x^2 + 36$

$= \underbrace{(x^2 - 9)}\underbrace{(x^2 - 4)}$

$= \boxed{(x-3)(x+3)(x+2)(x-2)}$

x^4 | 36
$x^2 \cdot x^2$ | $1 \cdot 36$, $2 \cdot 18$, $3 \cdot 12$, $4 \cdot 9$

$9x^2$, $4x^2$

It may help to let $y = x^2$ and then rewrite:

$(x^2)^2 - 13x^2 + 36$

$(y)^2 - 13y + 36$

Check: $(x^2-9)(x^2-4)$
$= x^4 - 4x^2 - 9x^2 + 36$
$= x^4 - 13x^2 + 36$ ✓

Perfect Square Trinomials

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

Note: $(x+9)^2 = x^2 + 18x + 81$

$(x+9)(x+9)$
$= x^2 + 9x + 9x + 81$
$= x^2 + 18x + 81$

$(3x-8)^2$
$= 9x^2 - 48x + 64$

$(x-5)^2 = x^2 - 10x + 25$

$(x-5)(x-5) = x^2 - 5x - 5x + 25$
$= x^2 - 10x + 25$

Ex: Factor

$16x^2 - 56xy + 49y^2$

Try: $\boxed{(4x - 7y)^2}$

Check $(4x-7y)(4x-7y)$

$16x^2 - 28xy - 28xy + 49y^2$

$16x^2 - 56xy + 49y^2$ ✓

Ex: $4x^2 - 20xy + 9y^2$

Try $(2x - 3y)^2$

check $(2x-3y)(2x-3y)$

$4x^2 - 6xy - 6xy + 9y^2$

$4x^2 - 12xy + 9y^2$ No!

(+) same signs

sum of $20xy$

Start over: $4x^2 - 20xy + 9y^2$

$\boxed{(2x - 9y)(2x - y)}$

$4x^2$ | $9y^2$
$x \cdot 4x$ | $9y \cdot y$
$2x \cdot 2x$ | $3y \cdot 3y$

$2xy$

$18xy$

Check:

$4x^2 - 2xy - 18xy + 9y^2$

$4x^2 - 20xy + 9y^2$

OR $\boxed{(2x-y)(2x-9y)}$

Ex: Factor x^2+25

$\boxed{\text{Prime}}$

Factor x^2+24

$\boxed{\text{Prime}}$

From last time: x^2 + positive is always prime.

Note: x^2-25

$=\boxed{(x-5)(x+5)}$

Note: x^2-24

is $\boxed{\text{prime}}$

Why is x^2+25 prime?

Write as a trinomial: $x^2+0x+25$

(+) same signs, want a sum of 0

A sum of 0 is impossible, so it is prime

25

$\wedge$

$1\cdot 25$

$5\cdot 5$

7.1: Simplifying Rational Expressions

Recall: Rational Number: can be written as a ratio (fraction, quotient) of two integers.

Examples of rational numbers

$0, \frac{5}{2}, 3, \frac{2}{3}, 2\frac{3}{5}, 0.35, -\frac{1}{4}, 0.\bar{3}$

0: $\frac{0}{2}$ or $\frac{0}{1}$; 3: $\frac{3}{1}$; $2\frac{3}{5}$: $\frac{13}{5}$; 0.35: $\frac{35}{100}$; $0.\bar{3}$: $\frac{1}{3}$

Rational Expression: can be written as the ratio (fraction, quotient) of two polynomials.

Examples of rational expressions

$$\frac{x^2+4}{x^3-7}, \quad \frac{x^2-7x+1}{2x^2-8x+5}, \quad \frac{3}{x^2+5x}$$

To simplify a rational expression, we reduce the fraction by "canceling out" (dividing out) factors that appear in both numerator & denominator.

Ex:

$$\frac{8}{20} = \frac{4\cdot 2}{4\cdot 5} = \frac{4}{4}\cdot\frac{2}{5} = 1\cdot\frac{2}{5} = \boxed{\frac{2}{5}}$$

$$\frac{8}{20} = \frac{\cancel{4}^{1}\cdot 2}{\cancel{4}_{1}\cdot 5} = \boxed{\frac{2}{5}}$$

Ex: Simplify.

$$\frac{\cancel{24}^{3} x^4 y^3}{\cancel{32}_{4} x^5 y} = \boxed{\frac{3y^2}{4x}}$$

Ex: Simplify.

$$\frac{x^2+8x+15}{x^2+5x+6} = \frac{\cancel{(x+3)}(x+5)}{\cancel{(x+3)}(x+2)} = \boxed{\frac{x+5}{x+2}}$$

Restricted values:

$\boxed{x \neq -3, \; x \neq -2}$

$x+3 \neq 0 \qquad x+2 \neq 0$

To get the restricted values, look at the uncancelled version of the denominator. Throw out values that make the denominator 0.

Simplify.

Ex:
$$\frac{x-3}{x^2-7x+12} = \frac{\cancel{x-3}}{\cancel{(x-3)}(x-4)} = \boxed{\frac{1}{x-4}}$$

Note: $\frac{\cancel{8}^{1}}{\cancel{24}_{3}} = \boxed{\frac{1}{3}}$

Restricted values:
$x \neq 4$
$x \neq 3$

Ex: $$\frac{5x^4y^2 - 15x^3y^3}{10x^6y^3 + 40x^5y^4} = \frac{\cancel{5}^{1}x^3y^2(x - 3y)}{\cancel{10}_{2}x^5y^3(x + 4y)}$$

Scratchwork GCF: $5x^3y^2$

$5x^4y^2 - 15x^3y^3$

$5x^3y^2(x - 3y)$

Check:

$5x^4y^2 - 15x^3y^3$ ✓

$$= \frac{1\,(x-3y)}{2x^2y\,(x+4y)} = \boxed{\frac{x-3y}{2x^2y(x+4y)}}$$

Ex: If I had $\frac{x+5}{x+2}$, could I cancel x?

No! Why not? Try it with $x = 1$:

$$\frac{1+5}{1+2} = \frac{6}{3} = 2$$

If I cross off the 1's, it's wrong!

$$\frac{\cancel{1}+5}{\cancel{1}+2} \neq \frac{5}{2}$$

(you can only cancel factors, not terms)

Ex: Simplify. $\dfrac{x-2}{4-x^2}$.

$$\frac{x-2}{-x^2+4} = \frac{x-2}{-1(x^2-4)}$$

$$= \frac{\cancel{x-2}^{\,1}}{-1(x+2)\cancel{(x-2)}_{\,1}} = \frac{1}{-1(x+2)}$$

$$= \boxed{-\frac{1}{x+2}}$$

Could also do this (not recommended):

$$\frac{x-2}{4-x^2} = \frac{x-2}{(2+x)(2-x)} = \frac{x-2}{(2+x)(-1)(-2+x)}$$

$$= \frac{\cancel{x-2}^{\,1}}{(x+2)(-1)\cancel{(x-2)}_{\,1}} = \frac{1}{-1(x+2)}$$

$$= \boxed{-\frac{1}{x+2}}$$

7.2: Multiplying and Dividing Rational Expressions

Ex: $\frac{2}{3} \cdot \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \boxed{\frac{8}{15}}$

Ex: $\frac{\cancel{16}^{1}}{\cancel{15}_{5}} \cdot \frac{\cancel{21}^{7}}{\cancel{32}_{2}} = \boxed{\frac{7}{10}}$

Ex: Simplify

$$\frac{x^2+2x}{x^2-4} \cdot \frac{x^2-4x+4}{x^2-x}$$

$$= \frac{\cancel{x}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}(x-2)}{\cancel{x}(x-1)}$$

[Factor all numerators and denominators]

$$= \boxed{\frac{x-2}{x-1}}$$

Ex: $(x+2) \cdot \frac{5}{4x^2-16}$

$$= \frac{x+2}{1} \cdot \frac{5}{4(x^2-4)}$$

$$= \frac{\cancel{x+2}}{1} \cdot \frac{5}{4\cancel{(x+2)}(x-2)} = \boxed{\frac{5}{4(x-2)}} = \boxed{\frac{5}{4x-8}}$$

both correct

Division of Rational Expressions

Ex: $\frac{3}{4} \div \frac{5}{7}$

$= \frac{3}{4} \cdot \frac{7}{5}$

$= \boxed{\frac{21}{20}}$

[dividing by a number is equivalent to multiplying by its reciprocal]

Ex: Simplify

$$\frac{x^2+3x}{x+5} \div \frac{9-x^2}{3x-6}$$

$$= \frac{x^2+3x}{x+5} \cdot \frac{3x-6}{-x^2+9}$$

[rewrite as multiplication]

$$= \frac{x(x+3)}{x+5} \cdot \frac{3(x-2)}{-1(x^2-9)}$$

$$= \frac{x\cancel{(x+3)}}{x+5} \cdot \frac{3(x-2)}{-1\cancel{(x+3)}(x-3)} = \frac{3x(x-2)}{-1(x+5)(x-3)}$$

$$= \boxed{-\frac{3x(x-2)}{(x+5)(x-3)}}$$