3.2: Linear Equations in Two Variables (cont'd)

Note Title 2/5/2015

Example from last time: $-3x+2y=-4$

Choose an x and solve for y (or vice versa)

$x=4$

$$-3x+2y=-4$$
$$-3(4)+2y=-4$$
$$-12+2y=-4$$
$$+12 \quad +12$$
$$2y=8$$
$$\frac{2y}{2}=\frac{8}{2}$$
$$y=4$$

ordered pair: $(4,4)$

$x=1$

$$-3x+2y=-4$$
$$-3(1)+2y=-4$$
$$-3+2y=-4$$
$$+3 \quad +3$$
$$2y=-1$$
$$\frac{2y}{2}=\frac{-1}{2}$$
$$y=-\frac{1}{2}$$

ordered pair $\left(1,-\frac{1}{2}\right)$

$x=-2$

$$-3x+2y=-4$$
$$-3(-2)+2y=-4$$
$$6+2y=-4$$
$$-6 \quad -6$$
$$2y=-10$$
$$\frac{2y}{2}=\frac{-10}{2}$$
$$y=-5$$

ordered pair $(-2,-5)$

$y=3$

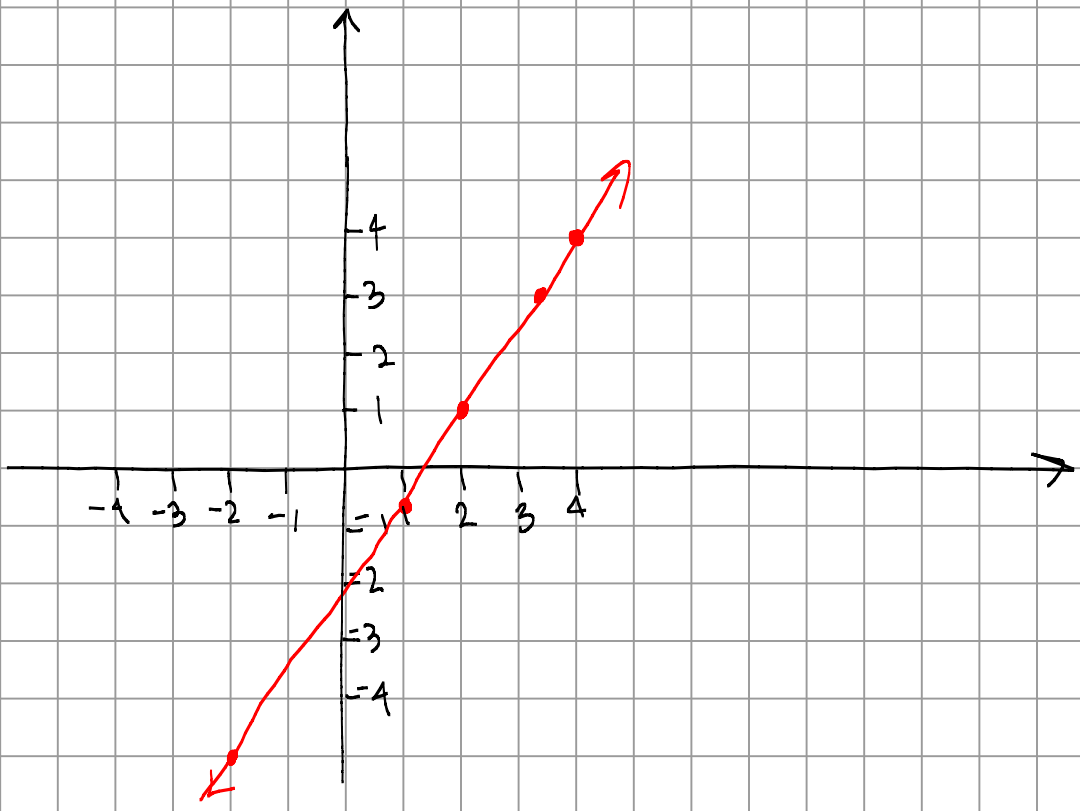
$$-3x+2y=-4$$
$$-3x+2(3)=-4$$
$$-3x+6=-4$$
$$-6 \quad -6$$
$$-3x=-10$$
$$\frac{-3x}{-3}=\frac{-10}{-3}$$
$$x=\frac{10}{3}$$
$$x=3\frac{1}{3}$$

ordered pair $\left(3\frac{1}{3},3\right)$

Graph on next page

Previous example cont'd.

ordered pairs: $(4,4)$
$(3\frac{1}{3},3)$
$(-2,-5)$
$(1,-\frac{1}{2})$



$x=2$ $-3x+2y=-4$

$-3(2)+2y=-4$

$-6+2y=-4$

$+6 \quad +6$

$2y=2$

$\frac{2y}{2}=\frac{2}{2}$

$y=1$

$(2,1)$

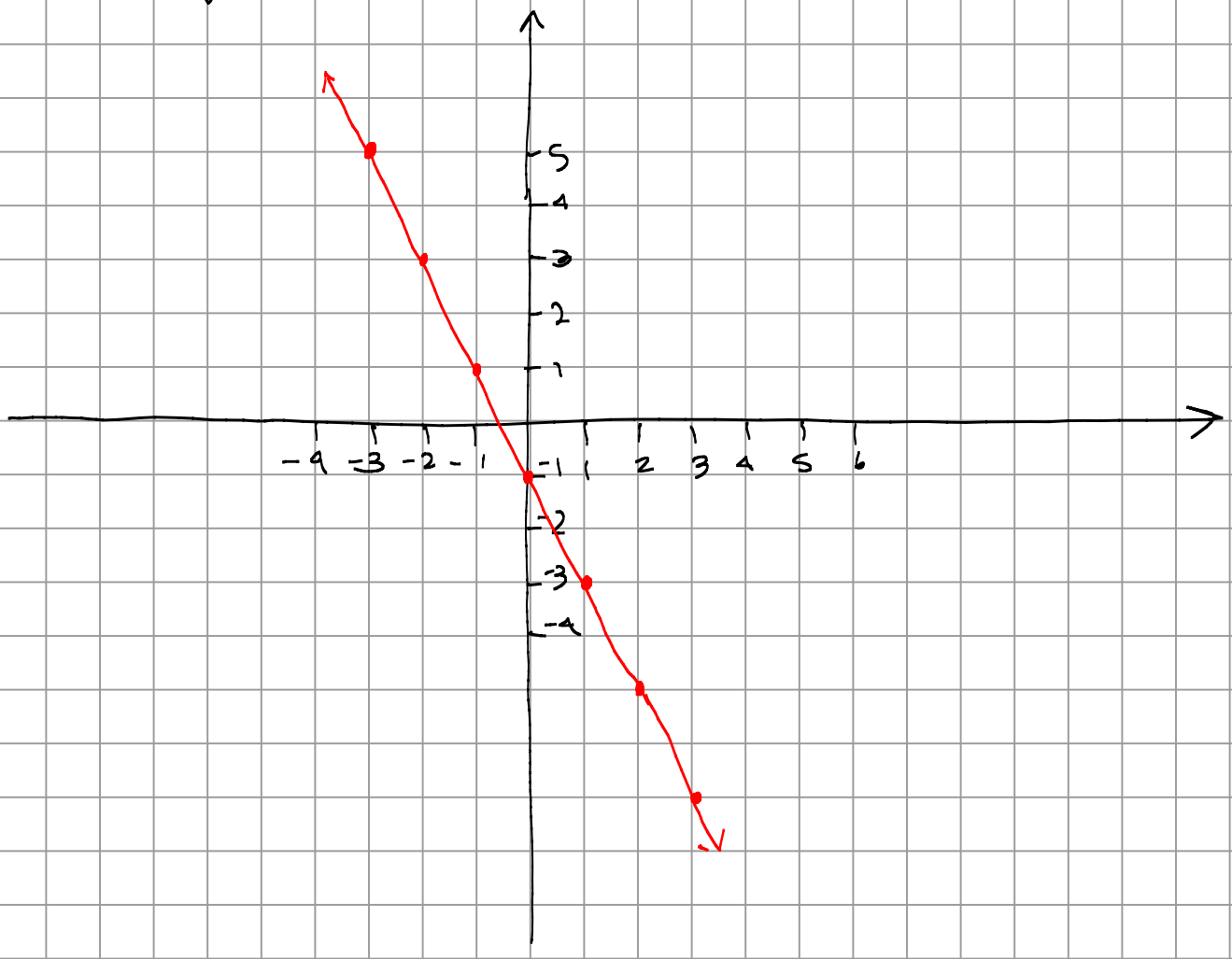
Example: Graph $2x + y = -1$ by solving for y 1st.

$$2x + y = -1$$
$$-2x \qquad -2x$$

Isolate y:

$$y = -2x - 1$$

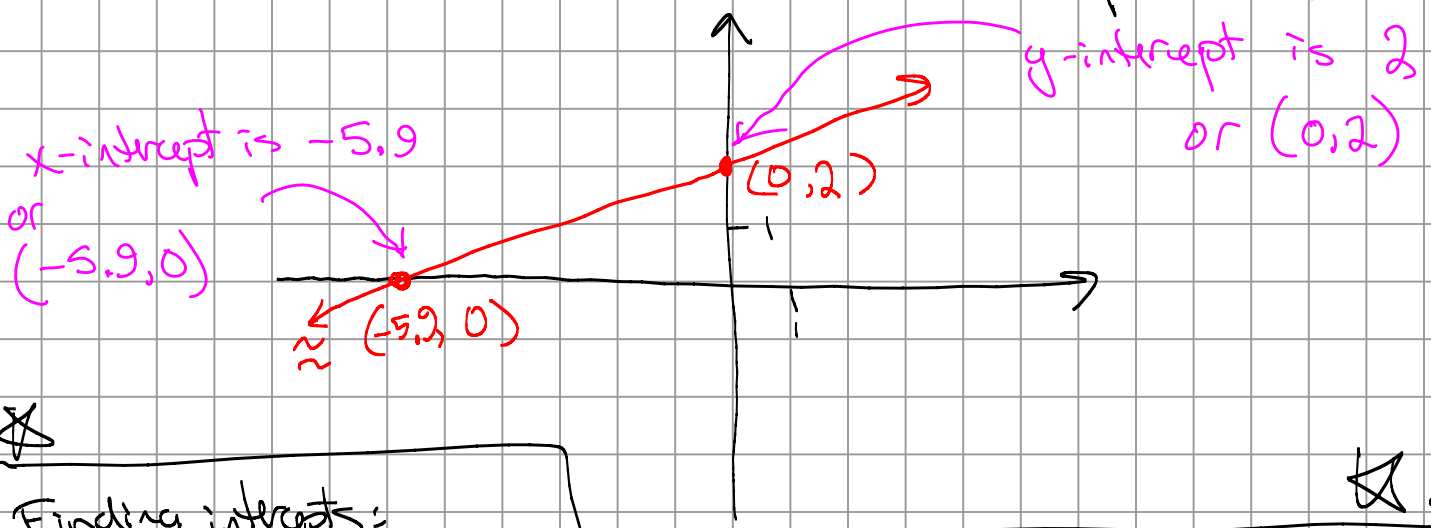
x	$y = -2x - 1$	Pair
0	$y = -2(0) - 1 = 0 - 1 = -1$	$(0, -1)$
1	$y = -2(1) - 1 = -2 - 1 = -3$	$(1, -3)$
2	$y = -2(2) - 1 = -4 - 1 = -5$	$(2, -5)$
3	$y = -2(3) - 1 = -6 - 1 = -7$	$(3, -7)$
-1	$y = -2(-1) - 1 = 2 - 1 = 1$	$(-1, 1)$
-2	$y = -2(-2) - 1 = 4 - 1 = 3$	$(-2, 3)$
-3	$y = -2(-3) - 1 = 6 - 1 = 5$	$(-3, 5)$



Intercepts

x-intercept: the x-coordinate of the point where the graph intersects the x-axis.
(Sometimes considered the ordered pair $(a, 0)$)

y-intercept: the y-coordinate of the point where the graph intersects the y-axis.
(Sometimes considered the ordered pair $(0, b)$.)



Finding intercepts:

* To find the x-intercept, set $y = 0$ and solve for x.
* To find the y-intercept, set $x = 0$ and solve for y.

Example: Find the intercepts and graph the line.

$$-2x + 4y = 12$$

Intercepts

$(-6, 0)$

$(0, 3)$

Find x-intercept:

Set $y = 0$

$$-2x + 4(0) = 12$$
$$-2x + 0 = 12$$
$$-2x = 12$$
$$\frac{-2x}{-2} = \frac{12}{-2}$$
$$x = -6$$

$(-6, 0)$

Find y-intercept:

Set $x = 0$

$$-2x + 4y = 12$$
$$-2(0) + 4y = 12$$
$$0 + 4y = 12$$
$$4y = 12$$
$$\frac{4y}{4} = \frac{12}{4}$$
$$y = 3$$

$(0, 3)$

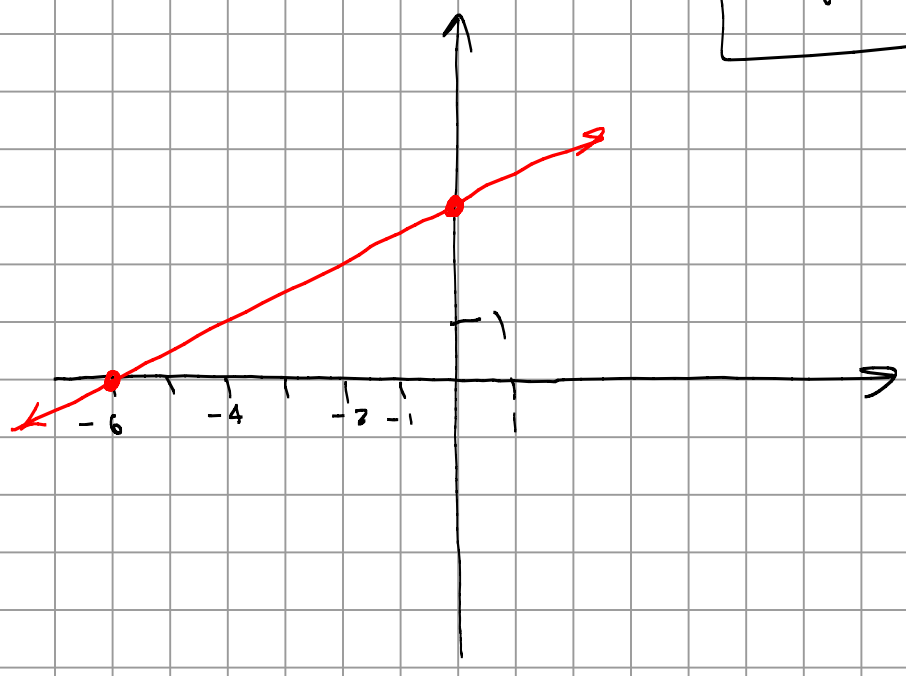
x-intercept: -6

y-intercept: 3

or

x-intercept: $(-6, 0)$

y-intercept: $(0, 3)$



Ex: Find the intercepts and graph the line.

$$x = -3y$$

Find x-intercept:
Set $y = 0$:

$x = -3(0)$
$x = 0$ x-intercept is 0
$(0, 0)$

x-intercept:	0 or (0,0)
y-intercept:	0 or (0,0)

Find y-intercept:
Set $x = 0$:

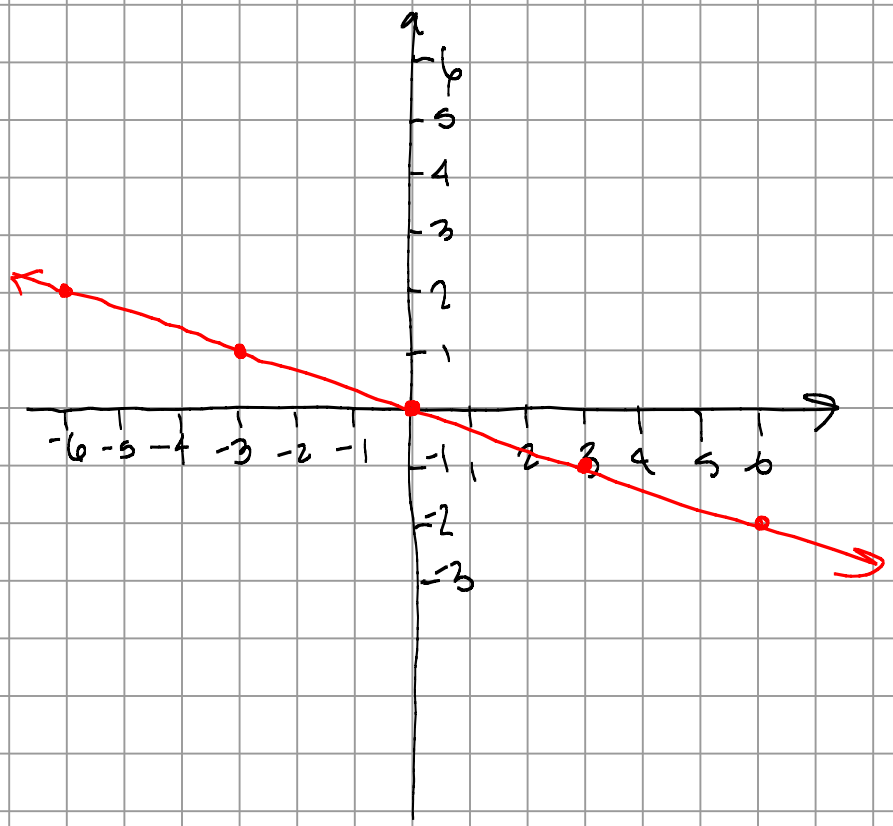
$x = -3y$
$0 = -3y$
$\frac{0}{-3} = \frac{-3y}{-3}$
$0 = y$

So y-intercept is 0. $(0, 0)$

The intercepts both gave me the same point, so I need to plot more points.

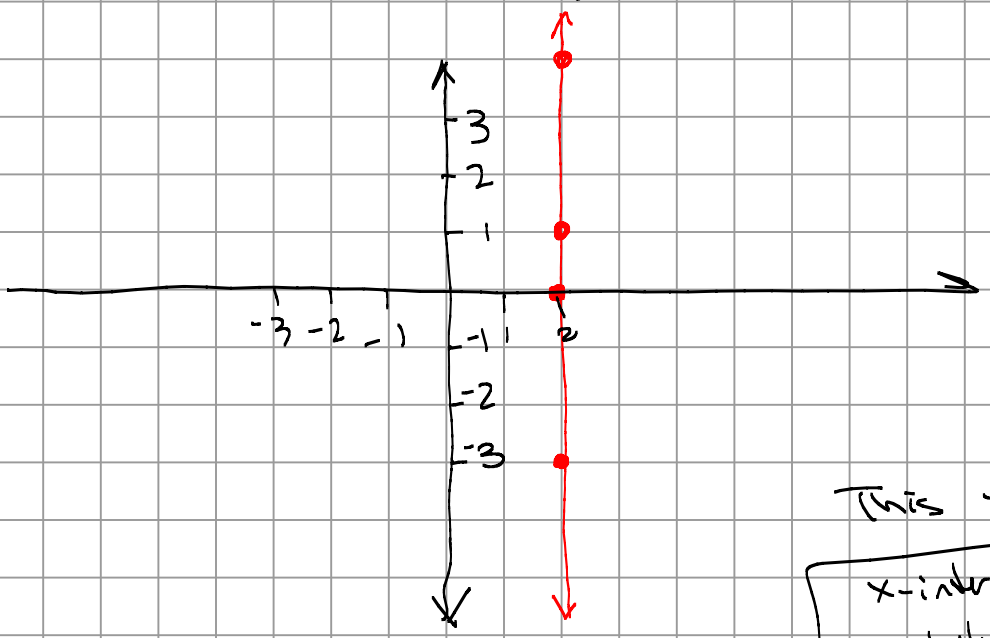
$x = -3y$

y	$x = -3y$	
1	$x = -3(1) = -3$	$(-3, 1)$
2	$x = -3(2) = -6$	$(-6, 2)$
−1	$x = -3(-1) = 3$	$(3, -1)$
−2	$x = -3(-2) = 6$	$(6, -2)$



Example: Graph the equation.

$x = 2$



Solutions

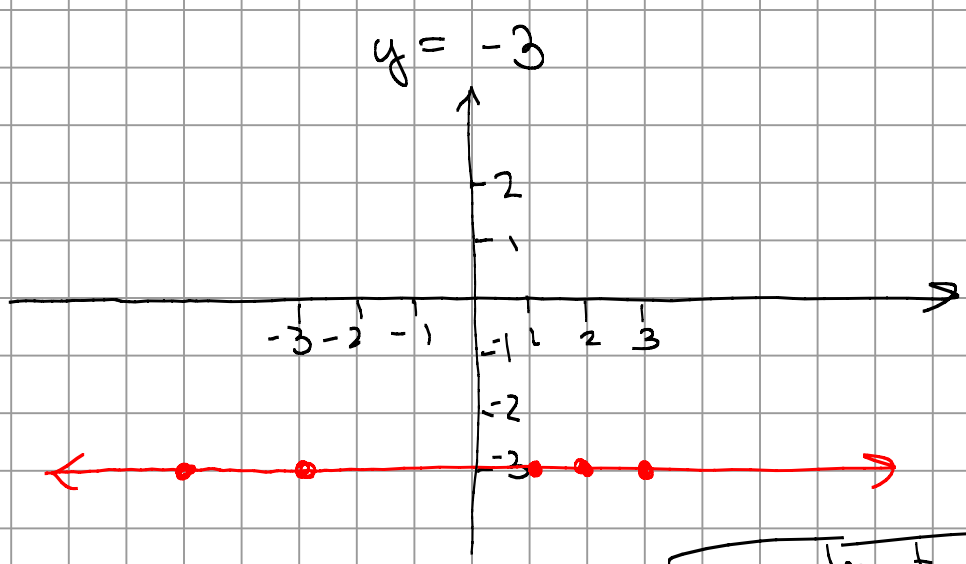
(2, 0)
(2, 1)
(2, 4)
(2, -3)
(2, 754)

This is a vertical line.

x-intercept: 2 or (2, 0)
y-intercept: does not exist

Ex: Graph the equation.

$y = -3$



Solutions

(1, -3)
(2, -3)
(3, -3)
(-3, -3)
(-5, -3)
(-21000, -3)

This is a horizontal line.

x-intercept: does not exist
y-intercept: -3 or (0, -3)

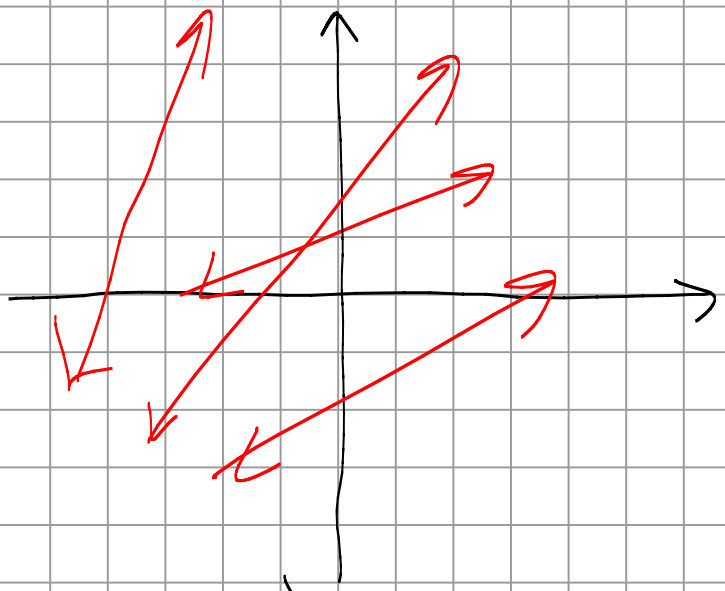
* Vertical lines take the form $x = k$, where k is a constant.
* Horizontal lines take the form $y = k$, where k is a constant.

3.3: Slope of a line

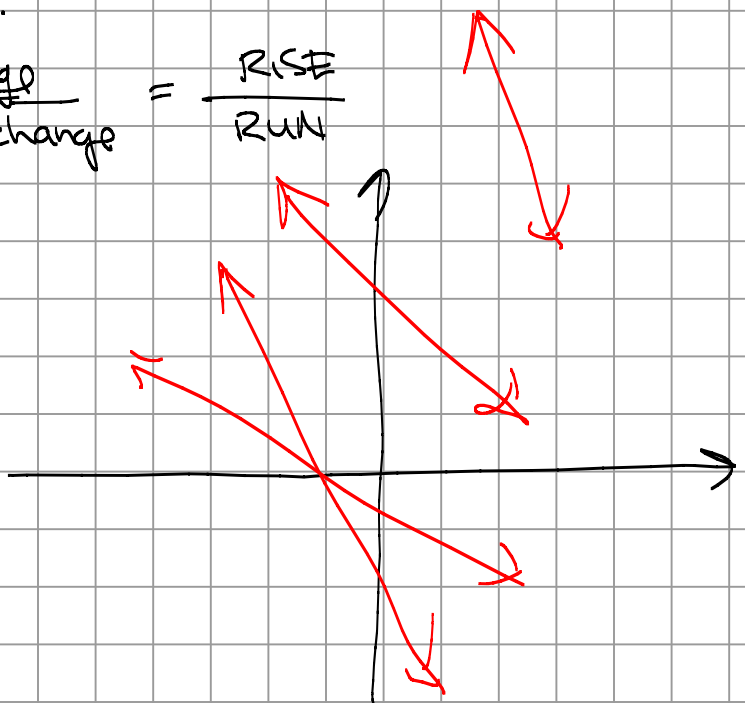
Slope: measures the "steepness" of a line

Definition: Slope $= m = \frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are points on the line.

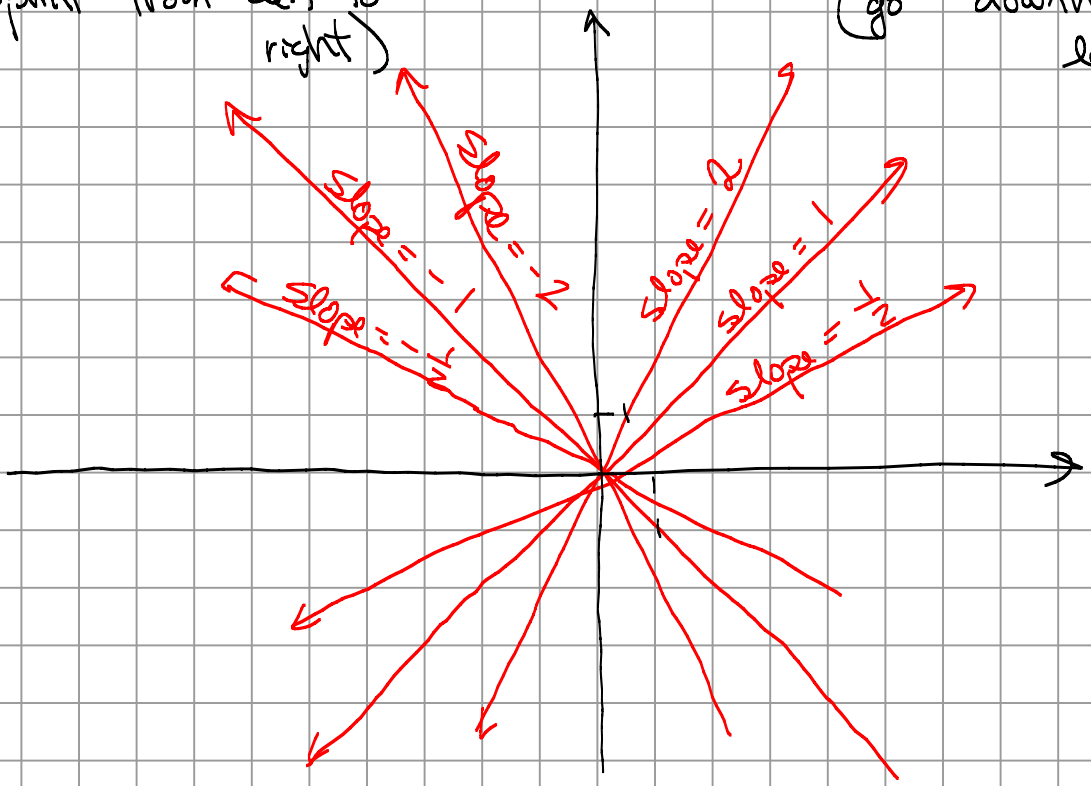
$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{RISE}}{\text{RUN}}$$



Positive Slopes
(go "uphill" from left to right)



Negative Slopes
(go "downhill" from left to right)



Example: Graph the line that passes through. $(1,-3)$ and has slope $\frac{2}{3}$.

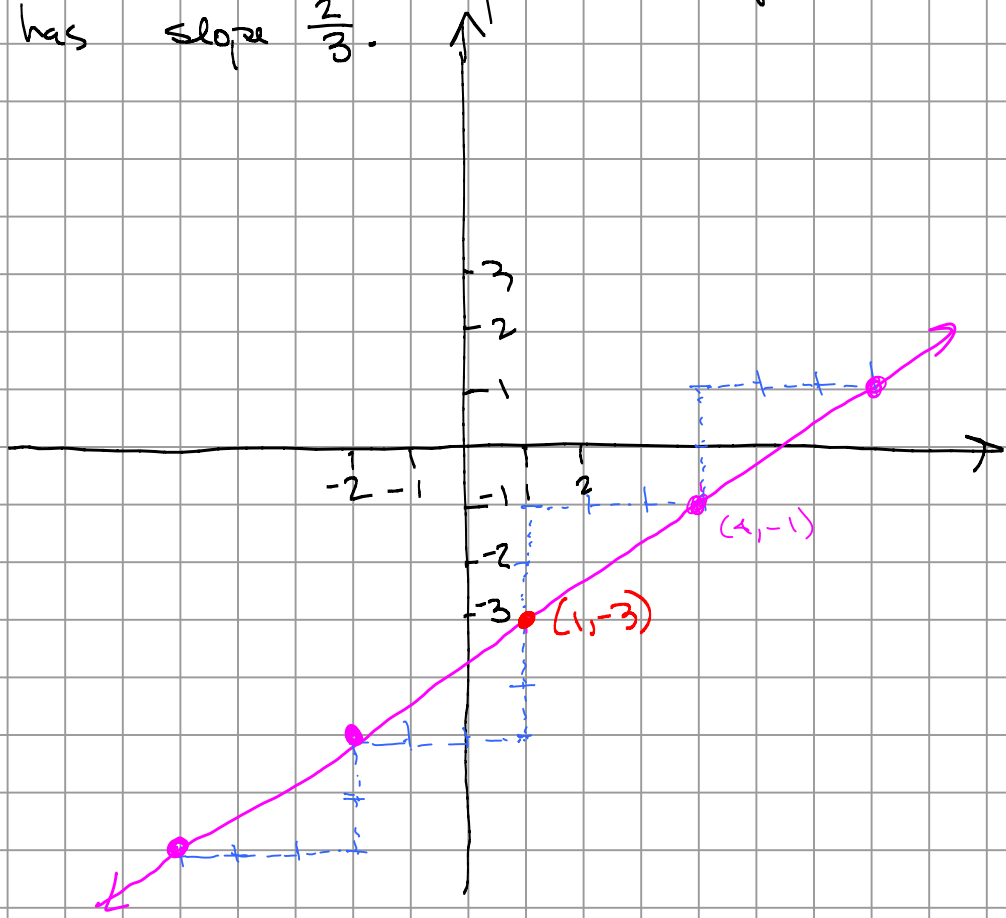
Slope: $m = +\frac{2}{3}$

positive slope: ↗

$\frac{\text{RISE}}{\text{RUN}} = \frac{2}{3}$

vertical change: 2

horizontal change: 3



Ex: Graph the line that passes through $(-2,-1)$ and has slope -3.

Slope: $m = -3 = -\frac{3}{1}$

Negative slope ↘

$\frac{\text{RISE}}{\text{RUN}} = -\frac{3}{1}$

vertical change: 3

horizontal change: 1