3.3: Slope of a Line (cont'd)

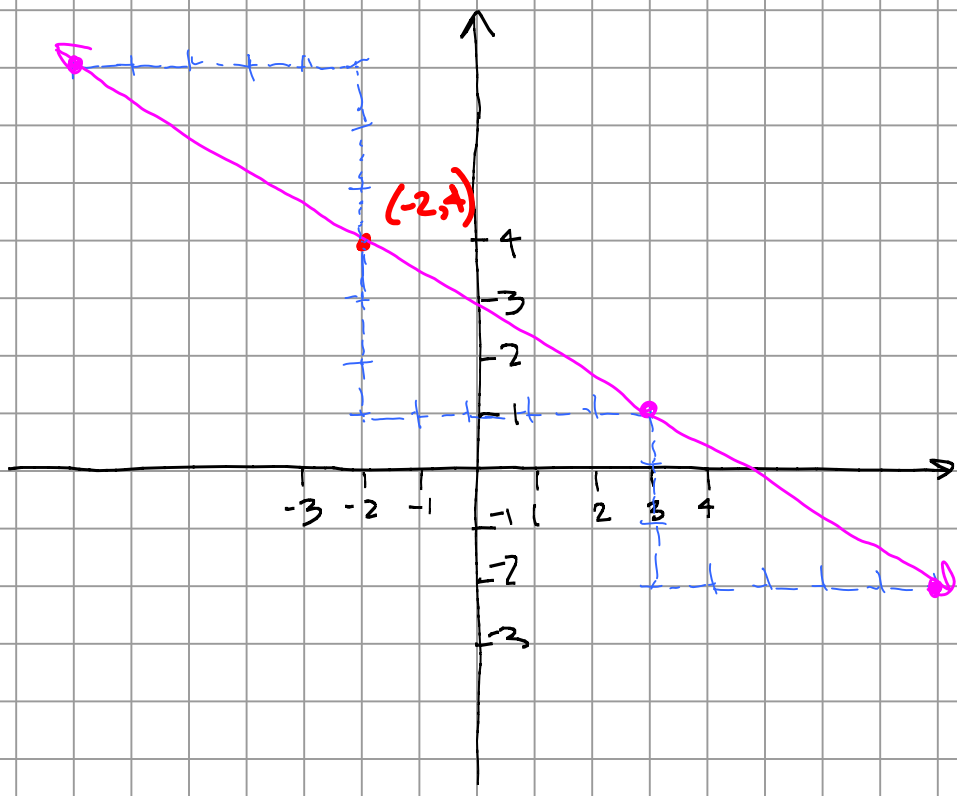
Note Title 2/5/2015

Example: Graph the line that has slope of $-\frac{3}{5}$ and passes through the point $(-2,4)$.

Slope: $m = -\frac{3}{5}$

vertical change ("rise"): 3

horizontal change ("run"): 5



Calculating the slope

Definition of Slope: Slope $= m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are points on the line.

Example: Find the slope of the line containing the points $(-3,2)$ and $(5,8)$.

(x_1, y_1) (x_2, y_2)

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{5-(-3)} = \frac{6}{5+3} = \frac{6}{8} = \boxed{\frac{3}{4}}$

We would get the same answer if we switched the order of the points:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-8}{-3-5} = \frac{-6}{-8} = \frac{6}{8} = \boxed{\frac{3}{4}}$$

Example: Find the slope of the line containing the points $(4, -6)$ and $(-1, -3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(y_2) - (y_1)}{(x_2) - (x_1)} = \frac{-3 - (-6)}{-1 - 4}$$

$x_1 = 4$
$x_2 = -1$
$y_1 = -6$
$y_2 = -3$

$$= \frac{-3 + 6}{-5} = \frac{3}{-5} = \boxed{-\frac{3}{5}}$$

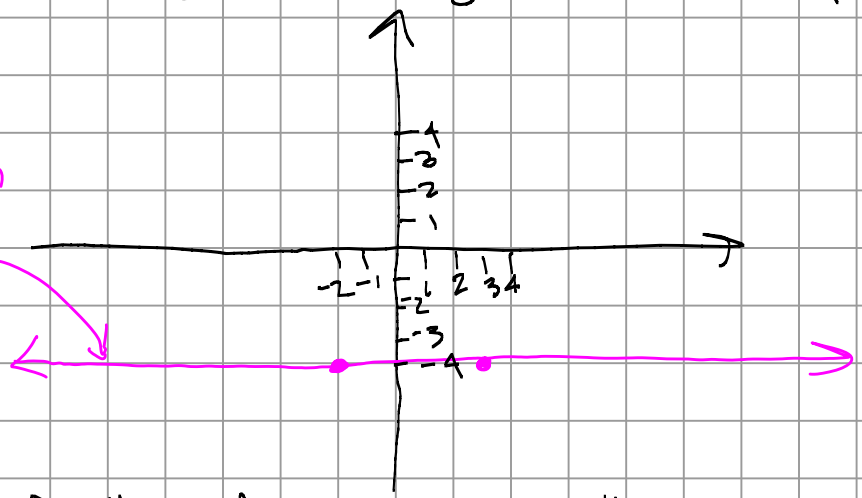
OR $m = \dfrac{-6 - (-3)}{4 - (-1)} = \dfrac{-6 + 3}{4 + 1} = \dfrac{-3}{5} = \boxed{-\dfrac{3}{5}}$

Ex: Find the slope of the line containing the points $(3, -4)$ and $(-2, -4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-4)}{-2 - 3} = \frac{-4 + 4}{-5} = \frac{0}{-5} = \boxed{0}$$

Let's graph it:

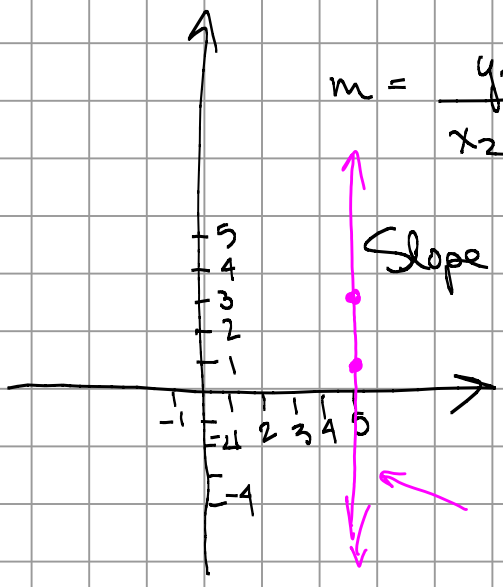
This is a horizontal line



Ex: Find the slope of the line containing the points $(5, 1)$ and $(5, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{5 - 5} = \frac{2}{0}$$

undefined (we cannot divide by 0)



Slope of this line is **undefined.**

This is a vertical line

Important:

* The slope of every horizontal line is 0.

* The slope of every vertical line is undefined.

Parallel and perpendicular lines

* Two lines are parallel if their slopes are equal.

* Two lines are perpendicular (intersect at a 90° angle / right angle) if their slopes are opposite reciprocals.

Ex: Suppose a line has slope $\frac{3}{7}$.

(a) what is the slope of a line parallel to it?

$\frac{3}{7}$

(b) what is the slope of a line perpendicular to it?

$-\frac{7}{3}$

Ex: Suppose a line has slope -4.

(a) what is slope of any line parallel to it?

-4

(b) what is slope of any line perpendicular to it?

$m_1 = -4 = -\frac{4}{1}$, then $m_2 = \boxed{+\frac{1}{4}}$

3.4: Slope-intercept Form of a Line

Standard Form of a Line: $Ax + By = C$ (where A and B are not both 0).

★ Slope-intercept Form of a line:

$$y = mx + b,$$

where m is the slope and b is the y-intercept or $(0, b)$

We can use slope-intercept form to graph.

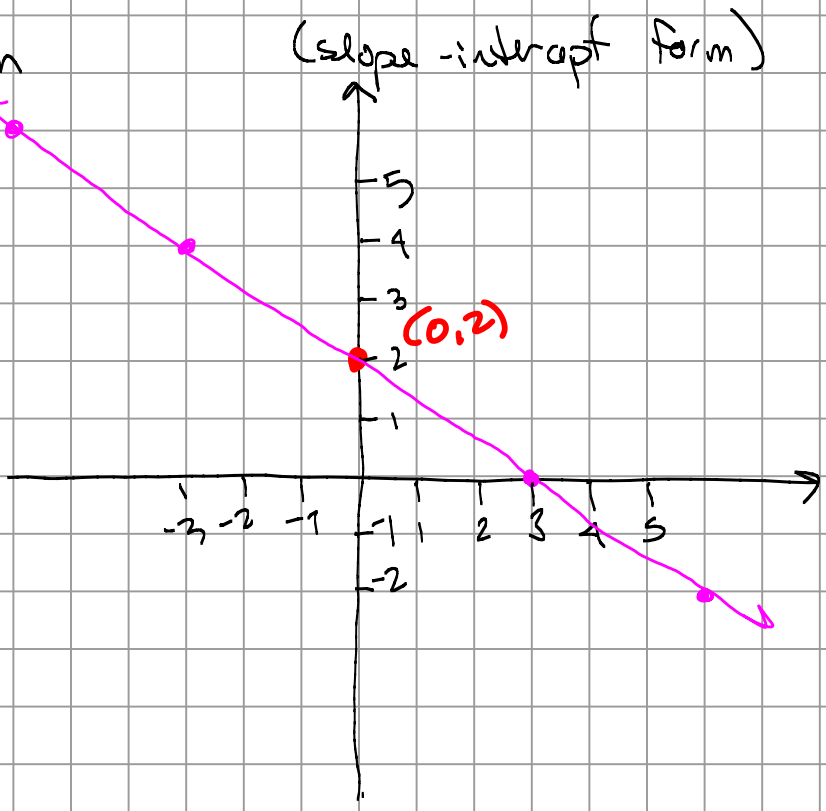
Ex: Graph the line $y = -\frac{2}{3}x + 2$.

This is in $y = mx + b$ form (slope-intercept form)

$$y = -\frac{2}{3}x + 2$$

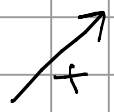
Slope: $m = -\frac{2}{3}$

y-intercept: $b = 2 \Rightarrow (0, 2)$

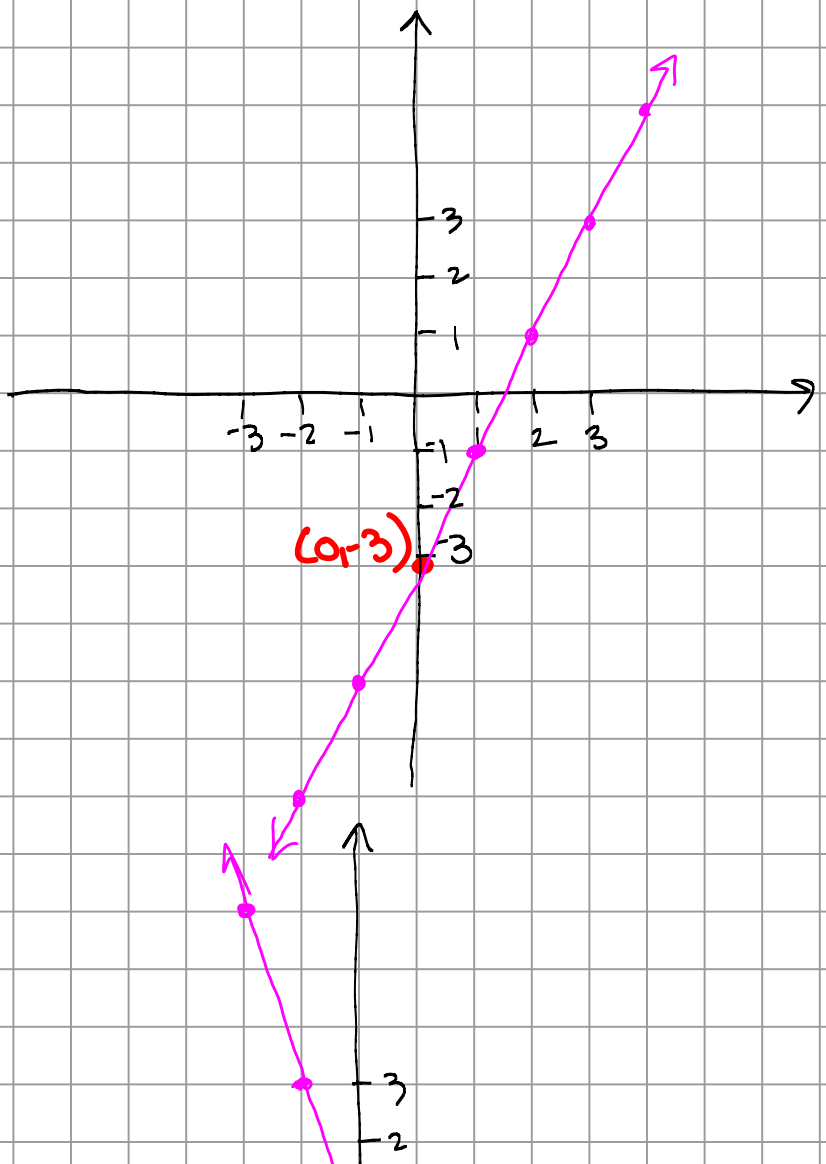


Ex: Graph $y = 2x - 3$

Slope: $m = 2 = +\frac{2}{1}$



y-intercept: $b = -3 \Rightarrow (0, -3)$

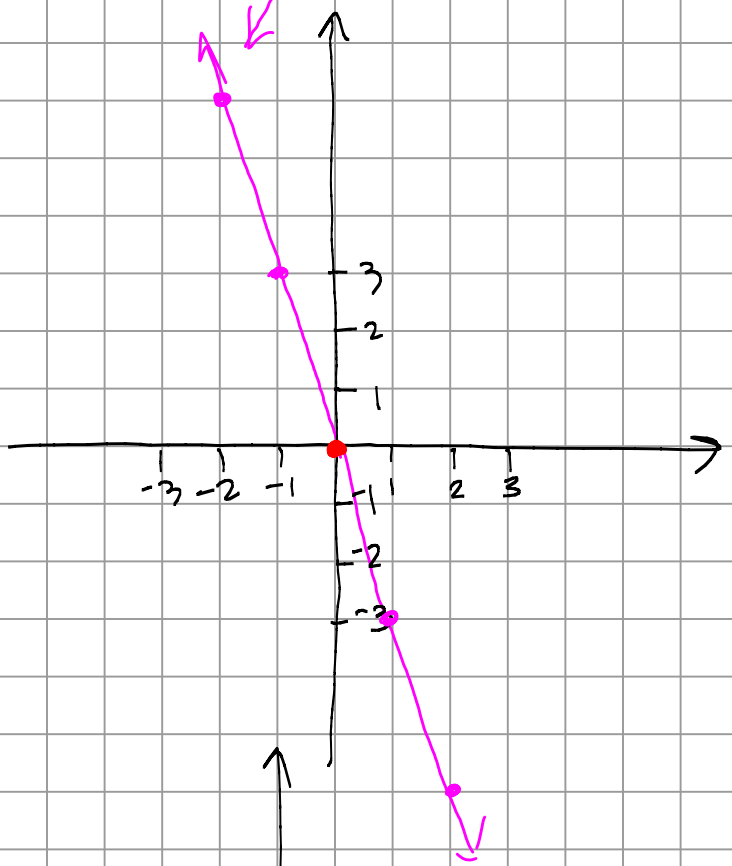


Ex: Graph $y = -3x$

Slope-intercept form: $y = -3x + 0$

Slope: $m = -3 = -\frac{3}{1}$

y-intercept: $0 \Rightarrow (0, 0)$

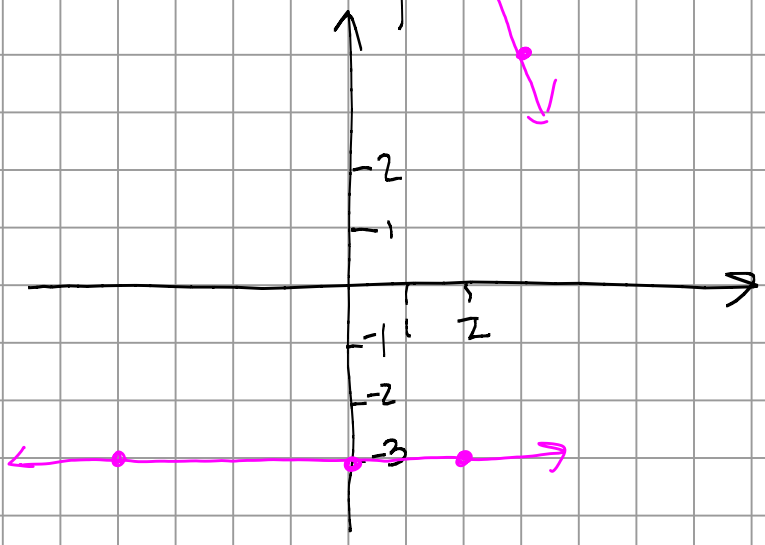


Ex: Graph $y = -3$

Find some pts $(0, -3)$

$(2, -3)$

$(-4, -3)$



Ex: Find the slope and the y-intercept of

$$3x+2y=9.$$

Write in $y=mx+b$ form:

$$3x+2y=9$$
$$-3x \qquad -3x$$
$$2y=-3x+9$$
$$\frac{2y}{2}=\frac{-3}{2}x+\frac{9}{2}$$

slope-intercept form: $\boxed{y=-\frac{3}{2}x+\frac{9}{2}}$

Slope: $m=-\frac{3}{2}$

y-intercept: $b=\frac{9}{2} \Rightarrow (0,\frac{9}{2})=(0,4\frac{1}{2})$

Ex: Write the line in slope-intercept form. Find the slope and y-intercept.

$$4x-3y=-18$$
$$-4x \qquad -4x$$
$$-3y=-4x-18$$
$$\frac{-3y}{-3}=\frac{-4x}{-3}-\frac{18}{-3}$$

$$\boxed{y=\frac{4}{3}x+6}$$ slope-int form

Slope: $m=\frac{4}{3}$

y-intercept: $b=6 \Rightarrow (0,6)$

Ex: What are the slope and y-intercept?

$y = -x$

$y = -1x + 0$

slope: $m = -1 = -\frac{1}{1}$

y-intercept: $b = 0 \Rightarrow (0, 0)$

4.1: Solving Linear Systems by Graphing

Note Title 7/5/2017

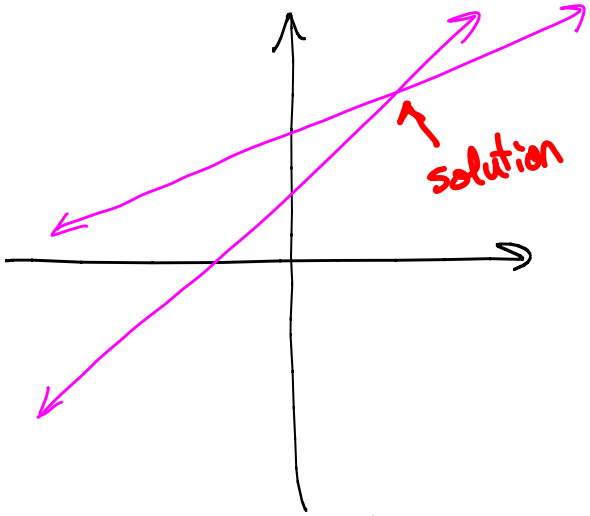
System of Equations: group of equations

Solution to a system (of equations): A set of values for the variables that makes **all** the equations **true**.

For us: we will work with systems of 2 equations in 2 variables (usually x and y)

Ex: $\begin{cases} 2x - 5y = 4 \\ x + 6y = 8 \end{cases}$ ⇒ graphs of these eqns are lines.

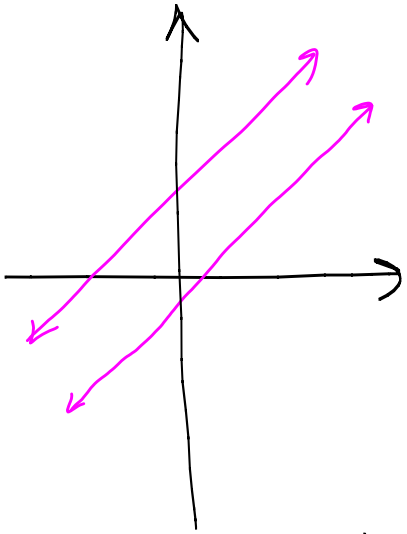
3 possible scenarios for a system of 2 linear eqns. (equations)



Independent System

One solution

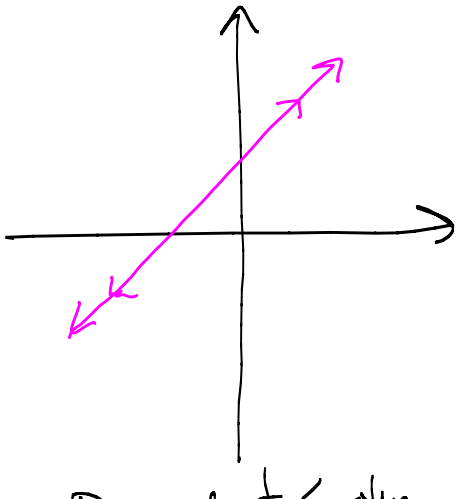
(Lines intersect at a single point.)



Inconsistent System

No Solution

(lines are parallel)



Dependent System

Infinitely many solutions

(lines are the same)

Ex: Is the ordered pair $(9,-2)$ a solution to the system?

$$2x+5y=8$$
$$3x-2y=23$$

Put $x=9$, $y=-2$ into $2x+5y=8$.

$$2(9)+5(-2)=8$$
$$18-10=8$$
$$8=8 \quad \checkmark \text{ true}$$

Put $x=9$, $y=-2$ into $3x-2y=23$

$$3(9)-2(-2)=23$$
$$27+4=23$$
$$31=23 \quad \text{False!}$$

No, $(9,-2)$ is not a solution to the system.

(it must work in both lines to be a solution of the system).

In Section 4.1, skip the "solve by graphing" problems.

(Do the problems where you check the point is a solution)

4.2: Solving Systems: The Substitution Method

Solving by substitution:

1) Solve 1 equation (your choice) for 1 variable (your choice).
2) substitute this result into the other equation.
3) Solve for the remaining variable.
4) Substitute this value into either equation and solve.
5) Check!

Ex: Solve the system by substitution.

$$\begin{cases} x - y = 9 \\ 2x - 11y = -18 \end{cases}$$

1) Solve $x - y = 9$ for x:

$$\begin{array}{rrr} x - y = & 9 & \\ +y & & +y \\ x = & y + 9 & \end{array}$$

2) substitute $x = y + 9$ into $2x - 11y = -18$

$$2(y+9) - 11y = -18$$

3) Solve for y:

$$2y + 18 - 11y = -18$$

$$\begin{array}{rrr} 18 - 9y = & -18 & \\ -18 & & -18 \end{array}$$

$$-9y = -36$$

$$\frac{-9y}{-9} = \frac{-36}{-9}$$

$$y = 4$$

4) Put $y = 4$ into $x - y = 9$

$$x - 4 = 9$$
$$+4 \quad +4$$
$$x = 13$$

Solution: $\boxed{(13, 4)}$

5) Check your answer:

$x = 13, y = 4$

$x - y = 9$

$13 - 4 = 9$

$9 = 9$ ✓

$2x - 11y = -18$

$2(13) - 11(4) = -18$

$26 - 44 = -18$

$-18 = -18$ ✓