4.2: Solving Systems by Substitution (cont'd)

Note Title 7/10/2017

(on 4.1) only do the problems where it asks you to decide whether a point is a solution – omit the "solve by graphing" problems

(omit # 27 - 35)

Omit all of 4.4

Example: solve the system by substitution.

$$\begin{cases} x+y=-1 \\ 3x-y=-3 \end{cases}$$

Step 1: Solve $x+y=-1$ for x

$$\begin{array}{rcl} x+y & = & -1 \\ -y & & -y \end{array}$$

$$x=-y-1$$

Step 2: Substitute $x=-y-1$ (or $x=-1-y$) into $3x-y=-3$. (must put it into the <u>other</u> eqn)

$$3(-y-1)-y=-3$$

$$-3y-3-y=-3$$

Step 3: solve for y

$$\begin{array}{rcl} -4y-3 & = & -3 \\ +3 & & +3 \end{array}$$

$$-4y=0$$

$$\frac{-4y}{-4}=\frac{0}{-4}$$

$$y=0$$

Step 4: Put $y=0$ into $x+y=-1$. (could put it into either equation)

$$x+0=-1$$

$$x=-1 \quad \text{solving for } x.$$

Solution: $\boxed{(-1,0)}$

<u>independent system</u> (one solution)

Step 5

<u>Check</u>:

$x+y=-1$

$x=-1, y=0 \Rightarrow -1+0=-1$

$-1=-1$ ✓

$3x-y=-3$

$x=-1, y=0 \Rightarrow 3(-1)-0=-3$

$-3-0=-3$

$-3=-3$ ✓

Ex: Solve by substitution.

$$\begin{cases} 5x - 12y = 1 \\ 6y - x = -2 \end{cases}$$

Solve $6y - x = -2$ for x:

$$6y - x = -2$$
$$-6y \qquad -6y$$
$$-x = -6y - 2$$
$$\frac{-x}{-1} = \frac{-6y}{-1} - \frac{2}{-1}$$
$$x = 6y + 2$$

Substitute $x = 6y+2$ into $5x - 12y = 1$:

$$5(6y+2) - 12y = 1$$
$$30y + 10 - 12y = 1$$
$$18y + 10 = 1$$
$$-10 \qquad -10$$
$$18y = -9$$
$$\frac{18y}{18} = \frac{-9}{18}$$
$$y = -\frac{1}{2}$$

Put $y = -\frac{1}{2}$ into $6y - x = -2$:

$$\frac{6}{1}\left(-\frac{1}{2}\right) - x = -2$$
$$-\frac{6}{2} - x = -2$$
$$-3 - x = -2$$
$$+3 \qquad +3$$
$$-x = 1$$
$$\frac{-x}{-1} = \frac{1}{-1} \Rightarrow x = -1$$

Check: $6y - x = -2$

$$\frac{6}{1}\left(-\frac{1}{2}\right) - (-1) = -2$$
$$-3 + 1 = -2$$
$$-2 = -2 \checkmark$$

$$5x - 12y = 1$$
$$5(-1) - \frac{12}{1}\left(-\frac{1}{2}\right) = 1$$
$$-5 + \frac{12}{2} = 1$$
$$-5 + 6 = 1$$
$$1 = 1 \checkmark$$

Solution:

$$\left(-1, -\frac{1}{2}\right)$$

4.3: Solving Systems using the Elimination Method (Addition)

Steps for solving by elimination (addition):

1) Choose a variable to eliminate. Multiply one or both equations by a strategic number, so that when I add the equations, a variable will be eliminated. (Look for the least common multiple of the variable's coefficients)

2) Add the equations (1 variable should disappear)

3) Solve for the remaining variable.

4) Either (a) Repeat Steps 1-3 for the other variable
   or (b) plug your value into either of the original eqns and solve.

5) Check your solution.

Example: Solve the system by elimination.

$$\begin{cases} -5x+2y=-6 \\ 10x+7y=34 \end{cases}$$

$$-5x+2y=-6 \xrightarrow{(2)} -10x+4y=-12$$

$$10x+7y=34 \longrightarrow 10x+7y=34$$

Add: $0x+11y=22$

$$11y=22$$

$$\frac{11y}{11}=\frac{22}{11}$$

$$y=2$$

Now, either do elimination a 2^{nd} time to get rid of y, or put $y=2$ into either of the original eqns and solve.

Putting $y=2$ into $10x+7y=34$.

$$10x+7(2)=34$$

$$10x+14=34$$

$$-14 \quad -14$$

$$10x=20$$

$$\frac{10x}{10}=\frac{20}{10}$$

$$x=2$$

Sol'n: $\boxed{(2,2)}$

Check: 1^{st} eqn: $-5x+2y=-6$

$$-5(2)+2(2)=-6$$

$$-10+4=-6$$

$$-6=-6 \checkmark$$

2^{nd} eqn: $10x+7y=34$

$$10(2)+7(2)=34$$

$$20+14=34$$

$$34=34 \checkmark$$

Ex: Solve by elimination (addition).

$$\begin{cases} 11x - 5y = 2 \\ 3x - y = 1 \end{cases}$$

$$11x - 5y = 2 \longrightarrow 11x - 5y = 2$$
$$3x - y = 1 \xrightarrow{(-5)} -15x + 5y = -5$$

Add: $-4x + 0y = -3$

$$-4x = -3$$
$$\frac{-4x}{-4} = \frac{-3}{-4}$$
$$x = \frac{3}{4}$$

Could put $x = \frac{3}{4}$ into one of the eqns;

OR do elimination again to find y.

$$11x - 5y = 2 \xrightarrow{(3)} 33x - 15y = 6$$
$$3x - y = 1 \xrightarrow{(-11)} -33x + 11y = -11$$

Add: $0 - 4y = -5$

$$-4y = -5$$
$$\frac{-4y}{-4} = \frac{-5}{-4}$$
$$y = \frac{5}{4}$$

$$\boxed{\text{Sol'n: } \left(\frac{3}{4}, \frac{5}{4}\right)}$$

Check: 1st eqn: $11x - 5y = 2$

$x = \frac{3}{4}, y = \frac{5}{4} \Rightarrow \frac{11}{1}\left(\frac{3}{4}\right) - \frac{5}{1}\left(\frac{5}{4}\right) = 2$

$$\frac{33}{4} - \frac{25}{4} = 2$$
$$\frac{8}{4} = 2$$
$$2 = 2 \checkmark$$

2nd eqn $3x - y = 1$

$x = \frac{3}{4}, y = \frac{5}{4} \Rightarrow \frac{3}{1}\left(\frac{3}{4}\right) - \frac{5}{4} = 1$

$$\frac{9}{4} - \frac{5}{4} = 1$$
$$\frac{4}{4} = 1$$
$$1 = 1 \checkmark$$

Ex: Solve the system.

$$\begin{cases} 6y - x = 2 \\ 3x - 18y = 5 \end{cases}$$

$-x + 6y = 2 \xrightarrow{(3)} -3x + 18y = 6$

$3x - 18y = 5 \longrightarrow 3x - 18y = 5$

Add: $0x + 0y = 11$

$0 = 11$ False!

By substitution:

Solved 1st eqn for x: $-x = 2 - 6y$

$x = -2 + 6y$

Put into $3x - 18y = 5$

$3(-2 + 6y) - 18y = 5$

$-6 + \cancel{18y} - \cancel{18y} = 5$

$-6 = 5$ ← False!

$+6 \quad +6$

$0 = 11$ False!

$\boxed{\text{No Solution}}$ $\boxed{\text{Inconsistent system}}$ (lines are parallel)

Ex: Solve the system.

$$2x - 10y = 6$$
$$-5x + 25y = -15$$

$$2x - 10y = 6 \xrightarrow{(5)} 10x - 50y = 30$$
$$-5x + 25y = -15 \xrightarrow{(2)} -10x + 50y = -30$$

Add: $0 + 0 = 0$

$0 = 0$ True!

$\boxed{\text{Dependent system; infinitely many solutions}}$

(lines are the same)

Summary: when solving by substitution or elimination

If you get an ordered pair solution, it's an independent system with one solution.

If both variables disappear:

* If we get a false statement (such as $0 = 11$ or $5 = -6$), the system is inconsistent and has no solution. (lines are parallel)

* If we get a true statement (such as $0 = 0$ or $30 = 30$), then the system is dependent and has infinitely many solutions. (lines are the same)

7.7: Ratios and Proportions

Ratios:

Ratio $1:2$, $\frac{1}{2}$ or 1 to 2

$3:1$, $\frac{3}{1}$ 3 to 1

16 to 24: $\frac{\cancel{16}^{2}}{\cancel{24}_{3}} = \frac{2}{3}$ or $2:3$

7.7 to 10: $\frac{7.7}{10} = \frac{7.7}{10}\frac{(10)}{(10)} = \boxed{\frac{77}{100}}$ or $77:100$

(Same as 0.77)

6 oz to 15 oz: $\frac{6\,\cancel{oz}}{15\,\cancel{oz}} = \frac{\cancel{6}^{2}}{\cancel{15}_{5}} = \frac{2}{5}$ or $2:5$.

$3\frac{1}{2}$ to $12\frac{1}{4} \Rightarrow \frac{3.50}{12.25}\left(\frac{100}{100}\right) = \frac{350}{1225} = \frac{350 \div 5}{1225 \div 5}$

$= \frac{70 \div 5}{245 \div 5} = \frac{\cancel{14}^{2}}{\cancel{49}_{7}} = \boxed{\frac{2}{7}}$

or $3\frac{1}{2}$ to $12\frac{1}{4} \Rightarrow$

$\frac{7}{2}$ to $\frac{49}{4} \Rightarrow \frac{\frac{7}{2}}{\frac{49}{4}} = \frac{\cancel{7}^{1}}{\cancel{2}_{1}} \cdot \frac{\cancel{4}^{2}}{\cancel{49}_{7}} = \boxed{\frac{2}{7}}$

Proportion

Examples from handout:

$$\frac{10 \text{ diamonds}}{6 \text{ opals}} = \frac{5 \text{ diamonds}}{3 \text{ opals}}$$

$$\frac{20 \text{ students}}{5 \text{ microscopes}} = \frac{4 \text{ students}}{1 \text{ microscope}}$$

$$\frac{2\frac{1}{4} \text{ cups flour}}{24 \text{ cookies}} = \frac{6\frac{3}{4} \text{ cups}}{72 \text{ cookies}}$$

Solving proportions:

$$\frac{5}{3} = \frac{40}{x}$$

$$5x = 40(3)$$

$$5x = 120$$

$$\frac{5x}{5} = \frac{120}{5}$$

$$x = 24 \qquad \boxed{\{24\}}$$

Note: If $\frac{a}{b} = \frac{c}{d}$, this is equivalent to $ad = bc$.
(sometimes called cross-multiplying)

Why? $\frac{a}{b} = \frac{c}{d}$ $\qquad$ ~~L~~CD: bd

may not be the least, but bd is a common Denominator

$$\frac{a}{\cancel{b}}\left(\frac{\cancel{b}d}{1}\right) = \frac{c}{\cancel{d}}\left(\frac{b\cancel{d}}{1}\right)$$

$$ad = bc$$

Ex: $\frac{y}{10} = \frac{13}{4}$

$$4y = 13(10)$$

$$4y = 130$$

$$\frac{4y}{4} = \frac{130}{4}$$

$$y = \boxed{\frac{65}{2}} \Rightarrow y = \boxed{32\frac{1}{2}} \text{ or } y = \boxed{32.5}$$

$$\begin{array}{r} 32 \\ 2\overline{)65} \\ \underline{6} \\ 05 \\ \underline{4} \\ 1 \end{array}$$

Ex: Solve.

$$\frac{1}{x} = \frac{3}{x-6}$$

$$1(x-6) = 3x$$

$$\begin{aligned} x - 6 &= 3x \\ -x \quad &\quad -x \end{aligned}$$

$$-6 = 2x$$

$$\frac{-6}{2} = \frac{2x}{2}$$

$$-3 = x$$

Sol'n: $\{-3\}$

Ex:

$$\frac{x+5}{7} = \frac{4x-1}{8}$$

$$8(x+5) = 7(4x-1)$$

$$\begin{aligned} 8x + 40 &= 28x - 7 \\ -8x \quad &\quad -8x \end{aligned}$$

$$\begin{aligned} 40 &= 20x - 7 \\ +7 \quad &\quad +7 \end{aligned}$$

$$47 = 20x$$

$$\frac{47}{20} = \frac{20x}{20}$$

$$\frac{47}{20} = x$$

Sol'n: $\left\{\frac{47}{20}\right\}$

Write the proportion & solve (from handout)

1) $$\frac{1 \text{ inch}}{8 \text{ ft}} = \frac{2\frac{7}{8} \text{ in}}{x}$$ Let x = wall length

$$x(1 \text{ inch}) = 8 \text{ ft}\left(2\tfrac{7}{8} \text{ in}\right)$$

$$\frac{x(1\ \cancel{\text{inch}})}{\cancel{\text{inch}}} = \frac{8 \text{ ft}\left(2\tfrac{7}{8}\ \cancel{\text{inch}}\right)}{\cancel{\text{inch}}}$$

$$x = 8 \text{ ft}\left(2\tfrac{7}{8}\right)$$

$$= \frac{\cancel{8}\text{ ft}}{1}\left(\frac{23}{\cancel{8}}\right) = \boxed{23 \text{ ft}}$$

2) $$\frac{627 \text{ mi}}{12.3 \text{ gallons}} = \frac{1250 \text{ mi}}{x}$$ x = # of gallons

$$\frac{x(627\ \cancel{\text{mi}})}{\cancel{\text{mi}}} = \frac{1250\ \cancel{\text{mi}}\,(12.3 \text{ gall})}{\cancel{\text{mi}}}$$

$$627x = 1250(12.3) \text{ gallons}$$

$$627x = 15\,375 \text{ gallons}$$

$$\frac{627x}{627} = \frac{15\,375}{627} \text{ gallons}$$

$$x \approx 24.52 \text{ gallons}$$

3) $\dfrac{1\frac{1}{2} \text{ cups milk}}{4 \text{ servings}} = \dfrac{4 \text{ cups milk}}{x}$ $\quad x = \#$ of servings

$$x\left(1\tfrac{1}{2}\right)\frac{\cancel{\text{cups}}}{\cancel{\text{cups}}} = 4\,\frac{\cancel{\text{cups}}}{\cancel{\text{cups}}}(4 \text{ servings})$$

$$x\left(\tfrac{3}{2}\right) = 16 \text{ servings}$$

$$\frac{x\left(\frac{3}{2}\right)}{3/2} = \frac{16 \text{ servings}}{3/2}$$

$$x = \frac{16}{1} \cdot \frac{2}{3} = \frac{32}{3} = \boxed{10\tfrac{2}{3} \text{ servings}}$$