

12.2: Hypothesis Tests for One Population Proportion

In a hypothesis test for one population proportion, we are testing the hypothesis that the population proportion p is equal to a benchmark value p_0 .

As long as $np \geq 5$ and $n(1-p) \geq 5$, we can assume that the sample proportion

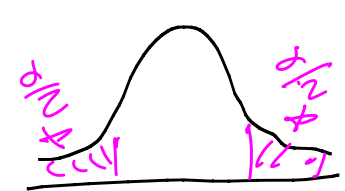
$\hat{p} = \frac{x}{n}$ is normally distributed. Therefore the critical values come from the z-distribution.

Hypothesis Testing for a Population Proportion:

Step 1: Determine the significance level α .

Step 2: Check that the assumptions are satisfied (simple random sample, $np \geq 5$ and $n(1-p) \geq 5$).

Step 3: Determine the null and alternative hypotheses.

Two-Tailed Test (most common)	Left-Tailed Test (rare)	Right-Tailed Test (rare)
$H_0 : p = p_0$ $H_a : p \neq p_0$	$H_0 : p = p_0$ $H_a : p < p_0$	$H_0 : p = p_0$ $H_a : p > p_0$
 Rejection Region	 Rejection Region	 Rejection Region

Note: One tailed tests assume that the scenario not listed ($p > p_0$ for a left-tailed test or $p < p_0$ for a right-tailed test) is not possible or is of zero interest.

Step 4: Use your α level and hypotheses, sketch the rejection region.

Step 5: Use a normal curve table (Table _____, on page _____) to determine the critical value for z associated with your rejection region. *(handout of z-table in our book)*

Step 6: Compute the test statistic $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.

std error: $\sqrt{\frac{p_0(1-p_0)}{n}}$

Step 7: Determine whether the value of z calculated from your sample (in Step 6) is in the rejection region.

- If z is in the rejection region, reject the null hypothesis.
- If z is not in the rejection region, do not reject the null hypothesis.

Step 8 State your conclusion.

Example 1: A local police department claims that over 70% of false alarms from home security systems are caused by human error (e.g., a house-sitter or child sets off the alarm by mistake). A random sample of 450 police files for false alarm calls are analyzed. Of the 450 false alarms in the sample, 324 were caused by human error. Does this sample provide evidence to support the police department's claim? Use a 95% confidence level.

Characteristic of interest: cause = human error

$$n = 450$$

$$x = 324$$

$$\hat{p} = \frac{x}{n} = \frac{324}{450} = 0.72$$

$$\hat{q} = 1 - \hat{p} = 1 - 0.72 = 0.28$$

$$H_0: p = 0.70$$

$$H_a: p > 0.70$$

$$\alpha = 0.05$$

From table, critical value is $z_{0.05} = 1.645$

Calculate test statistic for our sample

(z-score for our sample)

$$\text{std error: } \sigma_{\hat{p}} \approx \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.7(0.3)}{450}} \approx 0.021602$$

Careful! Use the benchmark proportion, not the sample proportion!

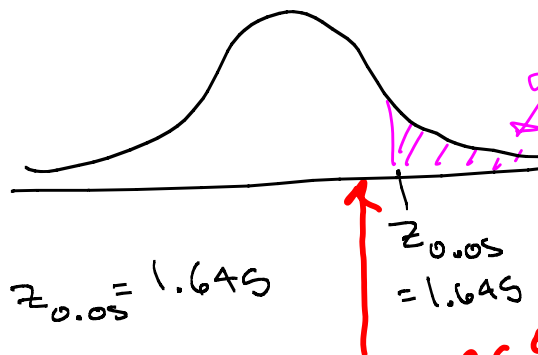
$$Z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.72 - 0.70}{0.021602} \approx 0.9258$$

Do not reject H_0 .

This sample does not provide evidence that the proportion of alarms caused by human error is over 70%.

Human error group: 324
Other causes: $450 - 324 = 126$

both ≥ 5 , so OK to use this procedure



$$\text{Benchmark} = 0.75$$

Example 2: According to genetic theory, if two red/sorrel Appaloosa-spotted horses are mated, there is a 0.75 probability that the foal will be a red/sorrel Appaloosa (otherwise, the baby will be red/sorrel with no Appaloosa spotting). Suppose a genetic researcher analyzed a sample of 200 foals in which both the sire and dam were red/sorrel Appaloosas, and found that 135 of the foals were red/sorrel Appaloosas. Does this sample provide evidence that the proportion of Appaloosa foals from all such matings differs from 0.75? Use a 90% confidence level.

<http://www.animalgenetics.us/Equine/CCalculator3.asp>

Characteristic: foal is red/sorrel Appaloosa

$$H_0: p = 0.75$$

$$H_a: p \neq 0.75$$

$$\alpha = 1 - 0.90 = 0.10$$

Sample info:

$$n = 200$$

$$\hat{p} = \frac{x}{n} = \frac{135}{200} = 0.675$$

Benchmark values

$$p_0 = 0.75$$

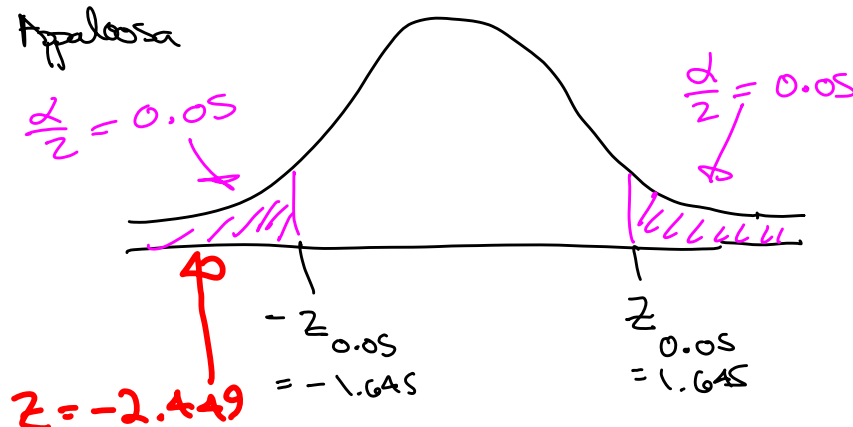
$$q_0 = 1 - 0.75 = 0.25$$

Calculate test statistic

$$Z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.675 - 0.75}{0.0306186} \approx -2.449$$

Critical value is $z_{0.05} = 1.645$

Our $Z = -2.449$ is in rejection region, so we reject H_0 . This sample provides evidence the proportion of red App babies differs from 0.75.



$$\begin{aligned} \text{Std error:} \\ \sigma_{\hat{p}} &= \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{p_0 q_0}{n}} \\ &= \sqrt{\frac{0.75(0.25)}{200}} \\ &\approx 0.0306186 \end{aligned}$$