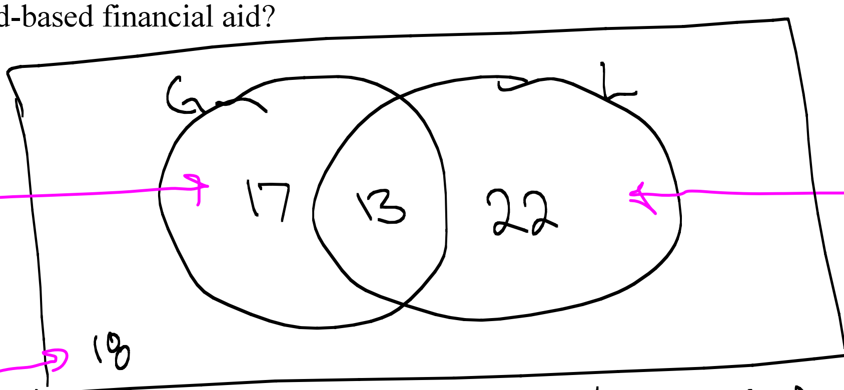
4.3: Some Rules of Probability

Example 1: Need-based financial aid for college students can take the form of grants (do not need to be repaid) or loans (must be repaid). Consider a group of 70 students in which 30 students received grants, 35 received loans, and 13 received both. How many of these students received need-based financial aid?

G: grants
L: loans

$30-13 = 17$



$35-13 = 22$

$70-17-13-22 = 18$

How many students rec'd financial aid?
$n(G \cup L) = 17+13+22 = \boxed{52}$

Notation: $n(A)$ means the number of elements in set A.

Addition Principle for Counting

For any two sets A and B,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

If A and B are mutually exclusive ($A \cap B = \varnothing$), then $n(A \cup B) = n(A) + n(B)$.

Mutually exclusive: no outcomes in common (also called *disjoint events*).

Prev. ex: $n(G \cup L) = n(G) + n(L) - n(G \cap L) = 30+35-13 = 65-13 = \boxed{52}$

Probability of unions and intersections:

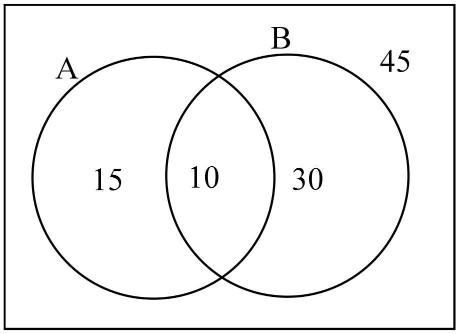
Probability of a Union of Two Events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If the two events are mutually exclusive (disjoint):

$$P(A \cup B) = P(A) + P(B)$$

Example 2: Assume that an equally likely sample space is described by the Venn diagram below.



100

$n(S) = 15 + 10 + 30 + 45 = 100$

$$P(B) = \frac{10+30}{100} = \frac{40}{100} = \boxed{0.40}$$

$$P(A \cap B) = \frac{10}{100} = \boxed{0.10}$$

$$P(B^c) = \frac{15+45}{100} = \frac{60}{100} = \boxed{0.60}$$

OR,

$$P(B^c) = 1 - P(B) = 1 - 0.40 = \boxed{0.6}$$

Complements:

Probability of a complement:
$P(E^C) = 1 - P(E)$
$P(E) = 1 - P(E^C)$

Example 1: Suppose that the probability of someone voting for a certain candidate is 0.46. What is the probability of not voting for the candidate?

$$P(E) = 0.46$$

$$P(E^c) = 1 - 0.46 = \boxed{0.54}$$

Example 2: Consider the data below, from the Congressional Research Service.
https://fas.org/sgp/crs/misc/RS20811.pdf

Table 1. Distribution of Household Money Income by Selected Income Class, 2012

Income Class	# of Households (in thousands)	% of Households
All Households	122,459	100.0
Less than $5,000	4,204	3.4
$5,000 to $9,999	4,729	3.9
$10,000 to $14,999	6,982	5.7
$15,000 to $19,999	7,157	5.8
$20,000 to $24,999	7,131	5.5
$25,000 to $29,999	6,740	5.4
$30,000 to $34,999	6,354	5.2
$35,000 to $39,999	5,832	4.8
$40,000 to $44,999	5,547	4.5
$45,000 to $49,999	5,254	4.4
$50,000 to $59,999	9,358	7.6
$60,000 to $69,999	8,305	6.8
$70,000 to $79,999	7,170	5.9
$80,000 to $89,999	5,969	4.9
$90,000 to $99,999	4,901	4.0
$100,000 to $124,999	9,490	7.7
$125,000 to $149,999	5,759	4.7
$150,000 to $199,999	6,116	5.0
$200,000 to $249,999	2,549	2.1
$250,000 and above	2,911	2.4
Median Income	$51,017	
Mean Income	$71,274	

Source: U.S. Census Bureau, 2012 Annual Social and Economic Supplement to the Current Population Survey.

E^c

$\geq \$20k$ (Part E)

E

D^c

a) What is the probability that a randomly selected household has an income of $100,000 or more?

$P(I > 100k) = 0.077 + 0.047 + 0.05 + 0.021 + 0.024$
$= \boxed{0.219}$

b) What is the probability that a randomly selected household has an income below $40,000?

$P(I < 40k) = 0.034 + 0.039 + 0.057 + 0.058 + 0.055 + 0.054$
$+ 0.052 + 0.048$

$= \boxed{0.397}$

~~c)~~ What is the probability that a randomly selected household has an income below $40,000?

d) What is the probability that a randomly selected household has an income below $250,000?

D: Income < \$250K. Use complement!

$P(D^c) = 0.024$. So $P(D) = 1 - 0.024 = \boxed{0.976}$

e) What is the probability that a randomly selected household has an income of $20,000 or more?

E: $I \geq \$20K$. $P(E^c) = 0.034 + 0.039 + 0.057 + 0.058 = 0.188$

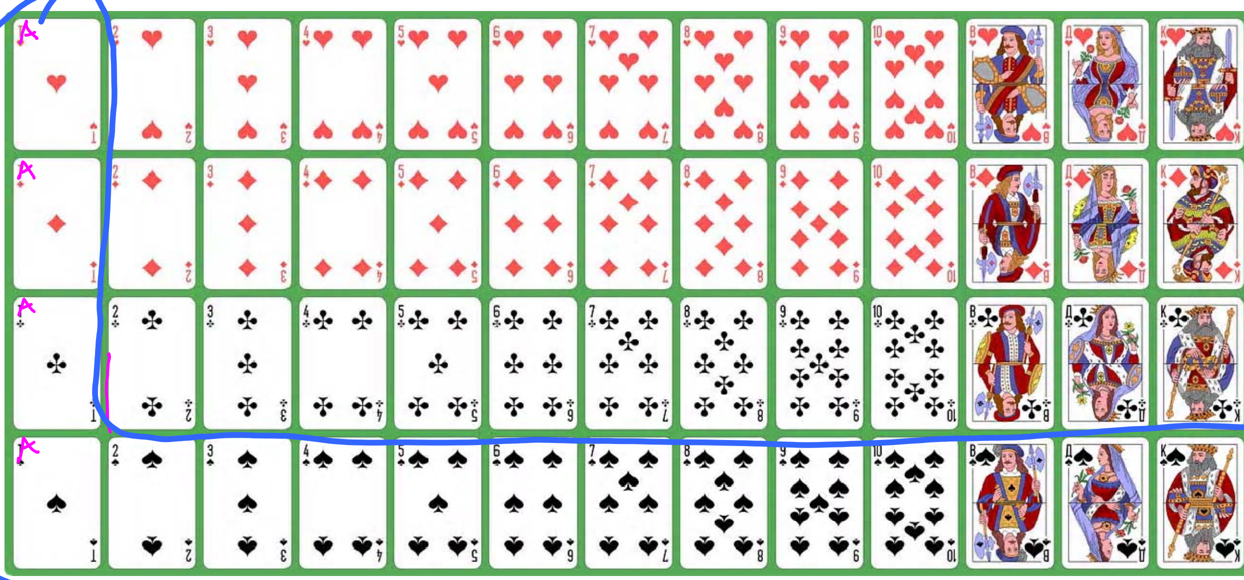
So $P(E) = 1 - P(E^c) = 1 - 0.188 = \boxed{0.812}$

f) Approximate the median household income.

omit

Example 3: Consider a standard deck of 52 cards.

Spades ∪ Aces



a) What is the probability that a randomly selected card is a spade or a heart?

$P(H \cup \text{Spade}) = P(H) + P(\text{Spade}) - P(H \cap \text{Spade})$

$= \frac{13}{52} + \frac{13}{52} - \frac{0}{52} = \frac{26}{52} = \boxed{\frac{1}{2}}$

b) What is the probability that a randomly selected card is a spade or an ace?

Spade ∪ Aces = {A, 2, 3, 4, …, 10, J, Q, K of spades, A ♢, A ♡, A ♧}

$P(\text{Spade} \cup \text{Aces}) = \frac{13+3}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$

or $n(\text{Spade} \cup \text{Aces}) = n(Sp) + n(A) - n(Sp \cap A) = 13 + 4 - 1 = 16$

$P(\text{Spade} \cup \text{Aces}) = \boxed{\frac{16}{52}}$

c) What is the probability that a randomly selected card is not a black face card?

BF = Black Face Cards

$n(BF) = 6$

$P(BF) = \frac{6}{52}$. So $P(BF)^c = 1 - \frac{6}{52} = \frac{52}{52} - \frac{6}{52} = \frac{46}{52} = \boxed{\frac{23}{26}}$

Note there are 46 cards that are not black face cards