5.2: The Mean and Standard Deviation of a Discrete Random Variable

Mean of a discrete random variable:

> The Mean (Expected value) of a Discrete Random Variable:
>
> Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of X is
>
> $$\mu = x_1 p_1 + x_2 p_2 + ... + x_n p_n.$$
>
> Suppose an experiment is repeated many times, and the values of X are recorded and then averaged. As the number of repetitions increases, the average value of X will become closer and closer to μ. For that reason, the mean is called the *expected value* of X.

Example 1: A probability distribution is given by the table below. Find the mean (the expected value of X).

x	3	4	5	6	7	8	9
$P(X = x)$	0.15	0.20	0.30	0.12	0.08	0.10	0.05

$$\mu = E(X) = 0.15(3) + 0.20(4) + 0.30(5) + 0.12(6) + 0.08(7) + 0.10(8) + 0.05(9)$$

$$= \boxed{5.28}$$

Example 2: Suppose that an organization sells 1000 raffle tickets for \$1 each. One ticket is for a gift basket worth \$200, and three tickets are for \$50 gift certificates to a restaurant. Find the expected net winnings for a person who buys one ticket.

X = net winnings for 1 ticket

Description	x	$P(X=x)$
Win \$200 basket	$200-1 = 199$	$\frac{1}{1000} = 0.001$
Win \$50 dinner	$50-1=49$	$\frac{3}{1000} = 0.003$
Lose (don't win a prize)	-1	$\frac{996}{1000} = 0.996$

Expected net winnings: $\mu = E(X) = 199(0.001) + 49(0.003) - 1(0.996)$

$= -\$0.65$

Example 3: Suppose the yearly premium for a car insurance policy is $2300 for a customer in a certain category. Statisticians for the insurance company have determined that a person in this category has a 0.007 probability of having an accident that costs the insurance company $100,000 and a 0.015 probability of having an accident that costs the insurance company $30,000. What is the expected value of the insurance policy to the customer? To the insurance company?

Let X = net value of policy to customer

Outcome	x	$P(X=x)$
Big accident	\$100000 − \$2300 = \$97 700	0.007
Little accident	30 000 − 2300 \$27 700	0.015
No accident	− \$2300	1 − 0.015 − 0.007 = 0.978

Sum: 1

$\mu = E(X) = \$97\,700(0.007) + \$27\,700(0.015) - \$2300(0.978)$

$= \boxed{-\$1150 \text{ (expected value to customer)}}$

Expected value to insurance company: \$1150

Standard deviation of a discrete random variable:

The Standard Deviation of a Discrete Random Variable:

Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the ~~mean~~ standard deviation of X is

$$\sigma = \sqrt{(x_1-\mu)^2 p_1 + (x_2-\mu)^2 p_2 + ... + (x_n-\mu)^2 p_n}$$
$$= \sqrt{\sum_{i=1}^{n} (x_i-\mu)^2 p_i}$$

variance: $\sigma^2 = \sum_{i=1} (x_i-\mu)^2 p_i$

Recall: Std dev = $\sqrt{\text{variance}}$

variance = $(\text{st. dev})^2$

and variance

Example 4: Calculate the mean and standard deviation of the probability distribution.

x	$P(X = x)$ [sometimes written $P(x)$]
0	0.11
1	0.32
2	0.43
3	0.10
4	0.04

Mean: $E(X) = \mu = 0(0.11) + 1(0.32) + 2(0.43) + 3(0.10) + 4(0.04) = \boxed{1.64 = \mu}$

x	$P(X=x)$	$(x_i - \mu)^2$	$(x_i - \mu)^2 p_i$
0	0.11	$(0-1.64)^2 = (-1.64)^2 = 2.6896$	$2.6896(0.11) = 0.295856$
1	0.32	$(1-1.64)^2 = (-0.64)^2 = 0.4096$	$0.4096(0.32) = 0.131072$
2	0.43	$(2-1.64)^2 = (0.36)^2 = 0.1296$	etc. 0.055728
3	0.10	$(3-1.64)^2 = (1.36)^2 = 1.8496$	0.18496
4	0.04	$(4-1.64)^2 = (2.36)^2 = 5.5696$	0.222784

Sum $= 0.8904$

Variance:
$\sigma^2 = 0.8904$

std. deviation:
$\sigma = \sqrt{0.8904}$
$\approx \boxed{0.9436}$

Example 5: Use the frequencies to construct a probability distribution for the random variable X, which represents the number of games bowled by customers at a bowling alley. Calculate the mean and standard deviation of X.

Number of Games	Frequency
1	37
2	45
3	29
4	12
5	4

See next page

Example 5: Use the frequencies to construct a probability distribution for the random variable X, which represents the number of games bowled by customers at a bowling alley. C mean and standard deviation of X.

X = # of games bowled

Number of Games	Frequency	Relative Freq
1	37	$\frac{37}{127} \approx 0.291$
2	45	45/127
3	29	29/127
4	12	12/127
5	4	4/127

$n = 127$

x	$P(X=x)$
1	37/127
2	45/127
3	29/127
4	12/127
5	4/127

Sum = 1

Mean: (expected value)

$$\mu = E(X) = 1\left(\frac{37}{127}\right) + 2\left(\frac{45}{127}\right) + 3\left(\frac{29}{127}\right) + 4\left(\frac{12}{127}\right) + 5\left(\frac{4}{127}\right)$$

$$= \frac{1}{127}\left(1(37) + 2(45) + 3(29) + 4(12) + 5(4)\right)$$

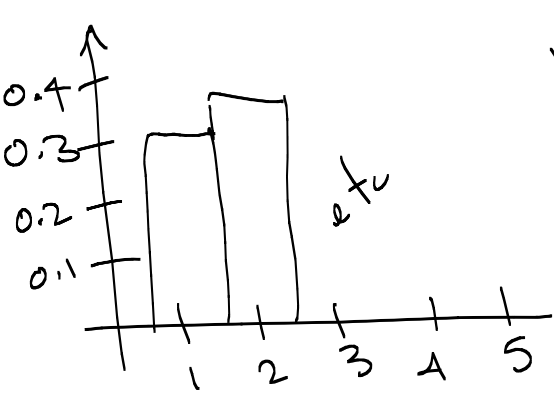
$$= \frac{282}{127} \approx \boxed{2.2205}$$

mean # of games

store calculator in A

x_i	$(x-\mu)^2$		$P(X=x)$	$(x-\mu)^2 p$
1	$(1-2.2205)^2$	≈ 1.48955	$\frac{37}{127} \approx 0.291$	0.433963
2	$(2-A)^2$	≈ 0.04861	$45/127 \approx 0.354$	0.017224
3	$(3-A)^2$	≈ 0.60766	29/127	0.138757
4	$(4-A)^2$	≈ 3.16672	12/127	0.299218
5	$(5-A)^2$	≈ 7.72577	4/127	0.243331

Sum $= \sigma^2 = 1.132493$



Variance: $\sigma^2 = 1.132493$

Std dev: $\sigma = \sqrt{\sigma^2} \approx \sqrt{1.132493}$

$\boxed{\sigma \approx 1.064}$ = std. dev. of the number of games