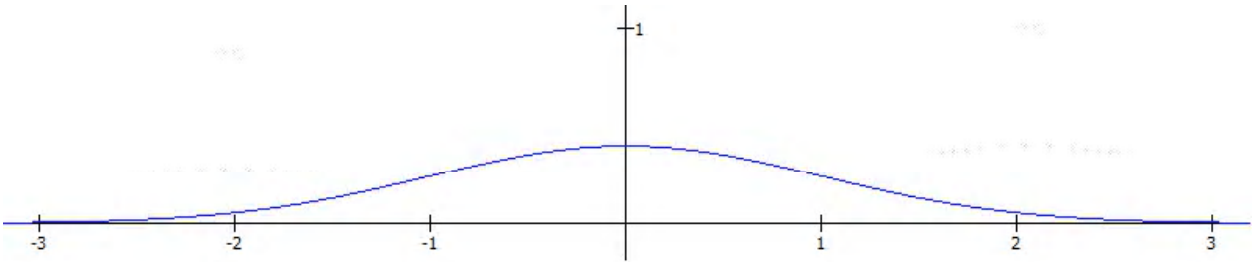
6.2: Areas Under the Standard Normal Curve

Areas under the standard normal curve:

One of the many remarkable things about normal curves is that for any normal curve, with any mean and standard deviation, the areas under the curve (and thus the associated probabilities and percentages) can be determined easily by using only one table.

The standard normal curve:

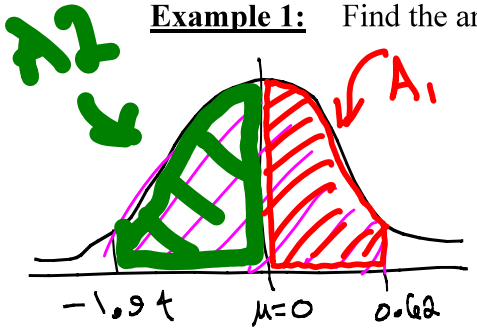


Properties of the Standard Normal Curve:

1. It is bell-shaped and symmetric about the line $x = 0$.
2. The standard normal distribution has mean 0 and standard deviation 1. ($\mu = 0$, $\sigma = 1$)
3. The area between the curve and the horizontal axis is always 1. (This corresponds to the fact that all the probabilities in a distribution must add up to 1.)
4. The curve approaches the *x*-axis asymptotically. (It gets closer and closer to the *x*-axis but never intersects it; the curve extends indefinitely in both directions).
5. The values on the *x*-axis can be thought of as *z*-scores.
6. Regardless of the shape,
   - 68.3% of the area is between 1 and -1.
   - 95.4% of the area is between 2 and -2.
   - 99.7% of the area is between 3 and -3.
7. The curve has inflection points at 1 and -1.

To determine areas under the curve and thus probabilities, we'll use a table. (See Table II in Appendix A, or inside the cover of your book, or use the handout.)

Example 1: Find the area under the standard normal curve between −1.94 and 0.62.



Area = $P(-1.94 < Z < 0.62) = A_1 + A_2 = 0.2324 + 0.4738 = 0.7062$

Find A_1: Look up $z = 0.62$ in table

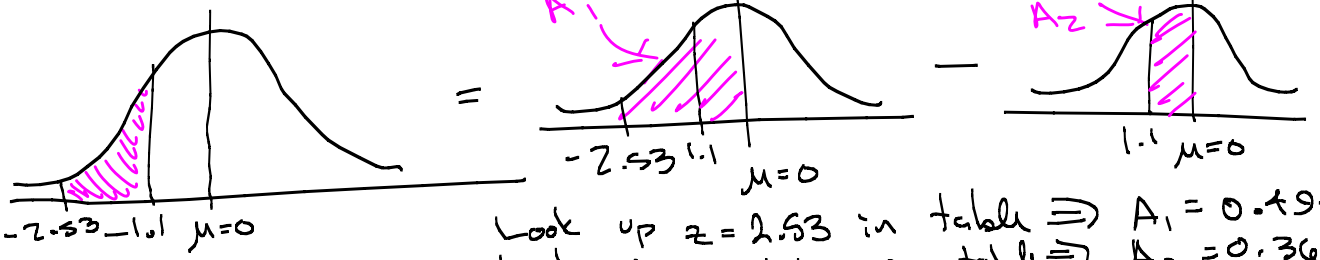
z	0.00	0.01	0.02
0.0			
0.1			
0.2			
0.3			
0.4			
0.5			
0.6			0.2324

From Table, $A_1 = 0.2324$

Look up $z = +1.94$ in Table.

From symmetry, $A_2 = 0.4738$

Example 2: Find the area under the standard normal curve between −1.1 and −2.53.

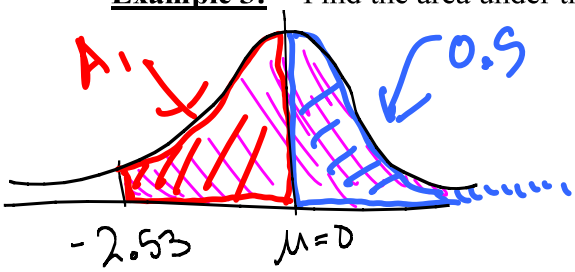


Look up $z = 2.53$ in table $\Rightarrow A_1 = 0.4943$

Look up $z = 1.10$ in table $\Rightarrow A_2 = 0.3643$

$P(-2.53 < z < 1.1) = A_1 - A_2 = 0.4943 - 0.3643 = 0.13$

Example 3: Find the area under the standard normal curve to the right of −2.53.



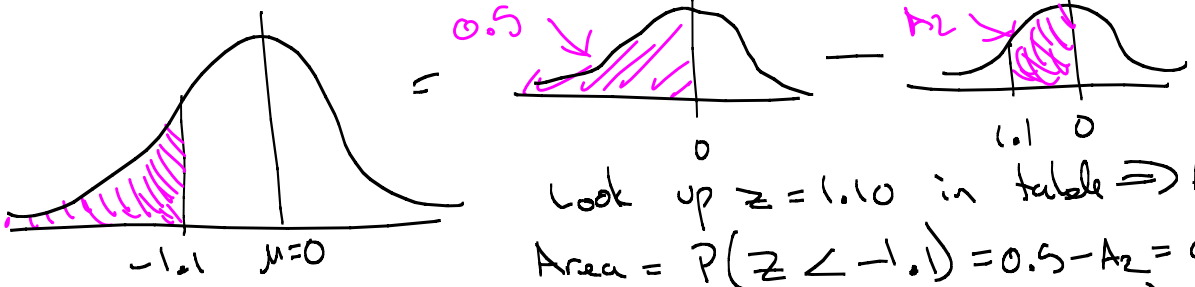
Look up $z = 2.53$ in table $\Rightarrow A_1 = 0.4943$

$P(Z > -2.53) = A_1 + 0.5$

$= 0.4943 + 0.5$

$= 0.9943$

Example 4: Find the area under the standard normal curve to the left of −1.1.



Look up $z = 1.10$ in table $\Rightarrow A_2 = 0.3643$

Area $= P(z < -1.1) = 0.5 - A_2 = 0.5 - 0.3643$

$= 0.1357$

Example 5: Find the area under the standard normal curve to the left of 2.08

What percent of area is left of −1.1?

13.57%

Finding a z-score associated with a certain area:

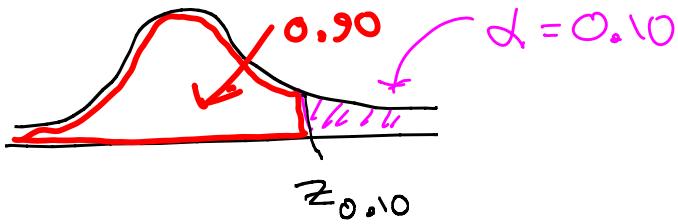
The symbol z_α indicates the z-score which has an area α to its right.

z_α α = greek letter alpha

Example 6: Find $z_{0.10}$, $z_{0.05}$, $z_{0.025}$, and $z_{0.0015}$.

(a) Find $z_{0.10}$

Here, $\alpha = 0.10$

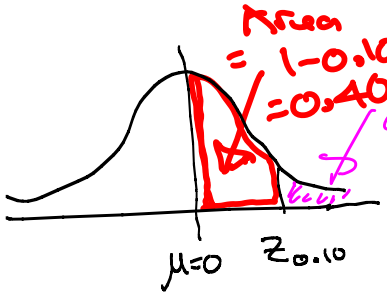


Using book table II (inside back cover)

Look up Area = 0.90. Closest area is 0.8997, corresponding to z-score of 1.28.

so $z_{0.10} \approx 1.28$

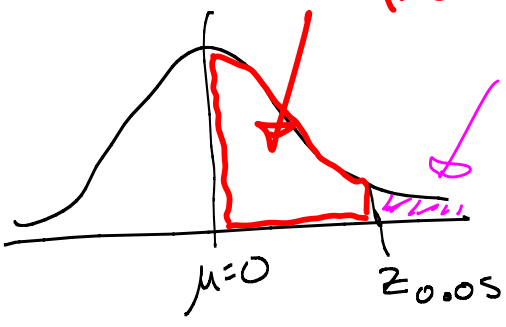
Using z-table from class:



Look up area = 0.40 in 1-page z-table. Closest area is 0.3997, again corresponding to z = 1.28.

So $z_{0.10} = 1.28$

(b) Find $z_{0.05}$



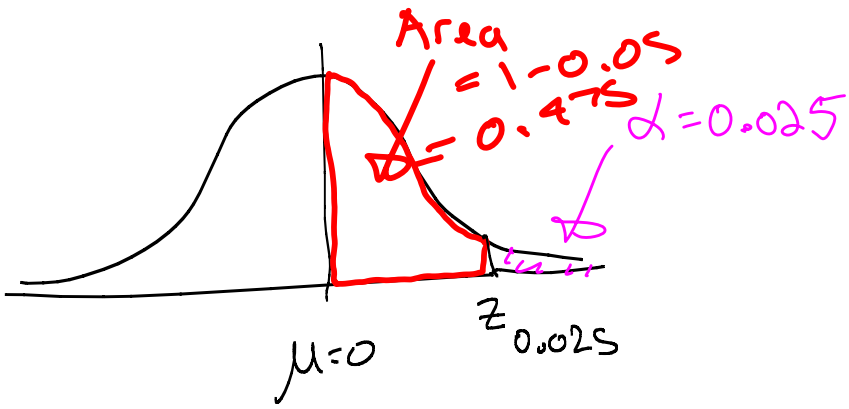
Look up area = 0.4

It's halfway between 0.4495 (corresponding to z = 1.64) and 0.4505 (corr. to z = 1.65)

Split the difference. $z_{0.05} = 1.645$

See next page

(c) Find $z_{0.025}$



Look up Area = 0.475

closest area is 0.4750, correspond to $z = 1.96$

So $z_{0.025} = 1.96$

(a super important value in statistics)