9.5: Hypothesis Tests for One Population Mean When σ is Unknown

In practice, when we are using a sample to make inferences about the population mean, it is rare for us to know the population standard deviation.

Instead, we must use the sample standard deviation, *s*, as a point estimate of the population standard deviation, σ.

When using *s* as an estimate for σ, we cannot use a *z*-test, because $\dfrac{\overline{x} - \mu}{\dfrac{s}{\sqrt{n}}}$ is not normally distributed.

The *t*-test for one population mean:

When using *s* as an estimate for σ, we use the Student *t*-distribution.

In order to use this procedure, we need to know (or be able to reasonably assume) that the variable of interest follows a normal distribution, or we must have a large sample size ($n \geq 30$).
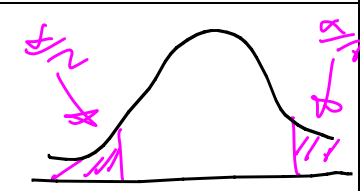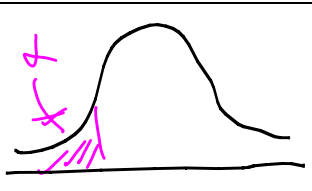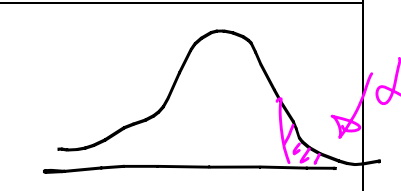
In addition, the sample should be randomly obtained, observations within the sample must be independent of one another. This means that if we have a sample size that is more than 5% of the population, we should multiply the standard error by a finite population correction factor, $\sqrt{\dfrac{N-n}{n-1}}$. (In this class, I do not anticipate that we will encounter this situation.)

Hypothesis Testing for a Population Mean:

Step 1: Determine the significance level α.

Step 2: Determine the null and alternative hypotheses.

Two-Tailed Test (most common)	Left-Tailed Test (rare)	Right-Tailed Test (rare)
$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$
Rejection Region	Rejection Region	Rejection Region

Note: One tailed tests assume that the scenario not listed ($\mu > \mu_0$ for a left-tailed test or $\mu < \mu_0$ for a right-tailed test) is not possible or is of zero interest.

Step 3: Use your α level and hypotheses, sketch the rejection region.

Step 4: Compute the test statistic $t = \dfrac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$.

Step 5: Use a table (Table IV, on page A-13) to determine the critical value for t associated with your rejection region. A-6 and A-7

Step 6: Determine whether the value of t calculated from your sample (in Step 3) is in the rejection region.
- If t is in the rejection region, reject the null hypothesis.
- If t is not in the rejection region, do not reject the null hypothesis.

Step 7: State your conclusion.

Conclusion in words:

This sample does not provide evidence that the Alaskans' mean temperature differes from 98.6°F.

9.5.3

Example 1: The normal human body temperature is widely accepted to be 98.6° F and can be assumed to follow a normal distribution. A medical researcher wants to know whether a certain geographical community of Native Alaskans has a mean body temperature of 98.6° F. A sample of 20 members of the Native Alaskan geographical community resulted in a mean body temperature of 98.3° F with a standard deviation of 0.7° F. Perform an appropriate hypothesis test at the 95% confidence level.

$H_0: \mu = 98.6°F$
$H_a: \mu \neq 98.6°F$

$\alpha = 1 - 0.95 = 0.05$

Sample info:

$n = 20$
$\bar{x} = 98.3°F$
$s = 0.7°F$
$df = n - 1 = 20 - 1 = 19$

→ does their body temperature differ from 98.6°F.

Find critical value of t:

$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

$\frac{\alpha}{2} = 0.025$

$-t_{0.025} = -2.093$

$t_{0.025} = 2.093$

-1.92
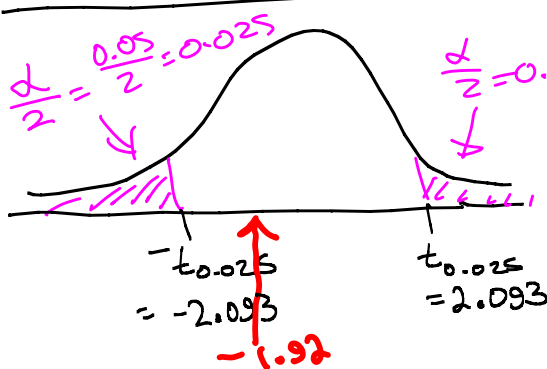
From t-table, for df = 19,
$t_{0.025} = 2.093$

this t is not in rejection region so Do not reject H_0

Calculate t for our sample:

standard error: $\sigma_{\bar{x}} \approx \frac{s}{\sqrt{n}} = \frac{0.7}{\sqrt{20}}$
≈ 0.1565

$t = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \approx \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{98.3 - 98.6}{0.1565} = -1.92$

Example 2: The average amount of lead in the blood of young children is 2 micrograms per deciliter (mcg/dL). A city has recently changed its water supply, and there have been widespread reports of increased lead levels in the water. A concerned doctor wants to dig into the city's medical records to find out whether the children in the city have blood lead levels above the average level of 2 mcg/dL. In a sample of 35 children, she found a mean lead level of 2.60 mcg/dL with a standard deviation of 1.9 mcg/dL. Perform an appropriate hypothesis test at the 95% confidence level.
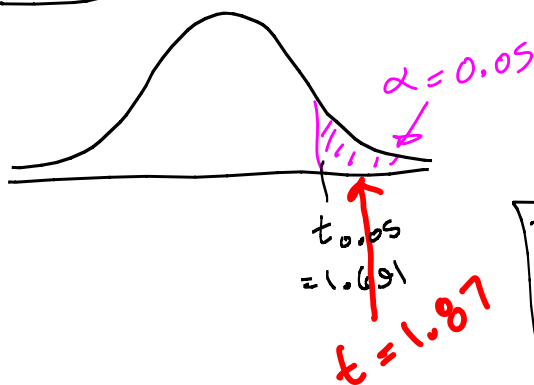
$H_0: \mu = 2 mcg/dL$
$H_a: \mu > 2 mcg/dL$ (right-tailed)

$\alpha = 1 - 0.95 = 0.05$

Find critical value of t:

$\alpha = 0.05$

$t_{0.05} = 1.691$

$t = 1.87$

From table, for df = 34,
$t_{0.05} = 1.691$

This sample provides evidence that kids' lead levels are higher than normal

Sample info: $n = 35$
$\bar{x} = 2.60$
$s = 1.9$
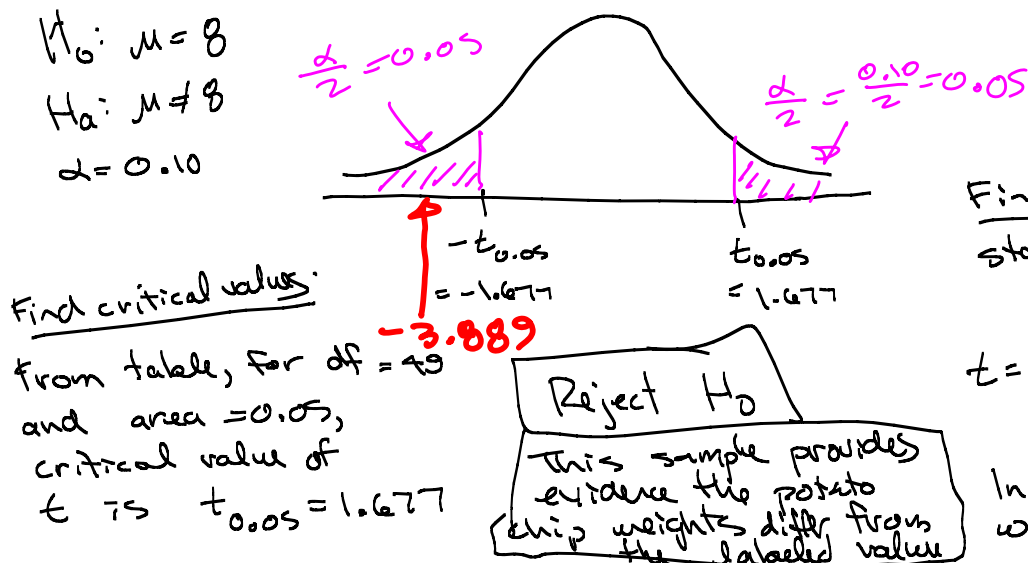
$n > 30 \Rightarrow$ ok to use t-test
$df = n - 1 = 34$

Find t for our sample

std. error: $\frac{s}{\sqrt{n}} = \frac{1.9}{\sqrt{35}} \approx 0.32116$
(store in calculator)

$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{2.60 - 2}{0.32116} = 1.87$

$t = 1.87$ is in rejection region, so we reject H_0

Example 3: Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. A customer (or a FDA analyst) buys 50 bags of chips, weighs them on a high-accuracy scale, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.

Sample info $\bar{x} = 7.89$

$H_0: \mu = 8$

$H_a: \mu \neq 8$

$\alpha = 0.10$

$\dfrac{\alpha}{2} = 0.05$

$\dfrac{\alpha}{2} = \dfrac{0.10}{2} = 0.05$

$-t_{0.05}$
$= -1.677$

$t_{0.05}$
$= 1.677$

-3.889

Find critical values:

from table, for df = 49 and area = 0.05, critical value of t is $t_{0.05} = 1.677$

Reject H_0

This sample provides evidence the potato chip weights differ from the labeled value
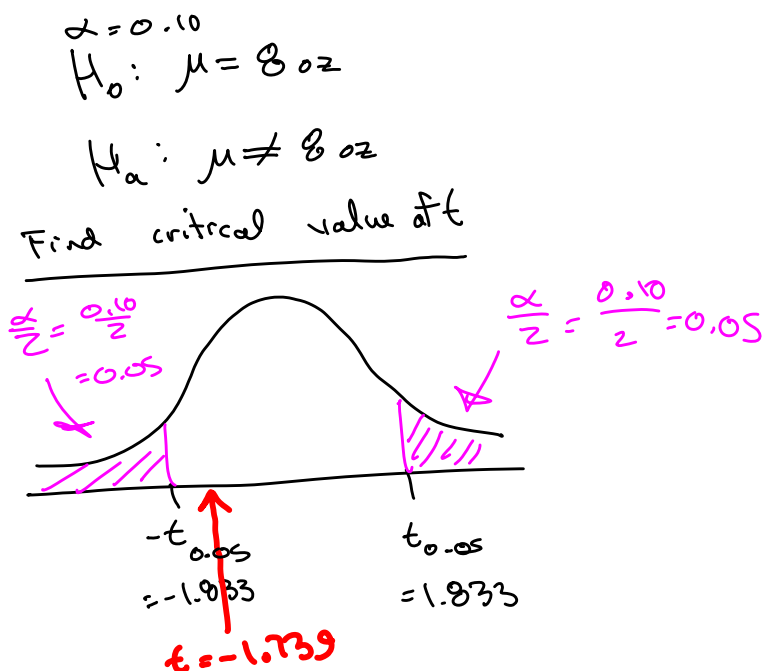
$s = 0.20$

$n = 50$

$df = n - 1 = 49$

Find t for our sample

std error: $\dfrac{s}{\sqrt{n}} = \dfrac{0.2}{\sqrt{50}} = 0.02828$

$t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{7.89 - 8}{0.02828}$

$= -3.889$

In the rejection region, so we Reject H_0

Example 4: Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. Again, a customer (or a FDA analyst) wonders whether the package size is accurate. This time, the analyist only buys 10 bags of chips, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.

$\alpha = 0.10$

$H_0: \mu = 8\,oz$

$H_a: \mu \neq 8\,oz$

Find critical value of t

$\dfrac{\alpha}{2} = \dfrac{0.10}{2} = 0.05$

$\dfrac{\alpha}{2} = \dfrac{0.10}{2} = 0.05$

$-t_{0.05}$
$= -1.833$

$t_{0.05}$
$= 1.833$

$t = -1.739$

From table, for df = 9,
$t_{0.05} = 1.833$

Do not reject H_0;

This sample does not provide evidence the mean weight differs from 8oz

Sample info:

$n = 10,$

$df = 10 - 1 = 9$

$\bar{x} = 7.89$

$s = 0.2$

To do this test, we would need to assume the weights are normally distributed (since sample size < 30)

Calculate t for our sample:

std error: $\sigma_{\bar{x}} \approx \dfrac{s}{\sqrt{n}} = \dfrac{0.2}{\sqrt{10}}$

≈ 0.06325

$t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{7.89 - 8}{0.06325} = -1.739$