Supplement: Basic Set Theory

Definition: A *set* is a well-defined collection of objects. Each object in a set is called an *element* of that set.

Examples of sets:

Set of positive integers
Set of students enrolled in this class
U.S. citizens over 6ft tall

Not sets:

all tall people
all cute dogs

Sets can be finite or infinite.

Examples of finite sets:

set of integers less than 500: {1,2,3,....., 498, 499}
students in this class
all people currently living

Examples of infinite sets:

Set of all integers
set of real numbers between 0 and 1
interval: (0,1)

Notation:

- We usually use capital letters for sets.
  We usually use lower-case letters for elements of a set.

- $a \in A$ means a is an element of the set A. $a \in A$
  $a \notin A$ means a is not an element of the set A. $a \notin A$

- The *empty set* is the set with no elements. It is denoted $\varnothing$. This is sometimes called the *null set*.
  $\varnothing$

- $S = \{x \mid P(x)\}$ means "S is the set of all x such that $P(x)$ is true". (called rule notation or set roster notation). $\{x \mid x < 2\}$ *set of all numbers x so that*
  $x < 2$
  so in interval notation, $(-\infty, 2)$

  Example: $S = \{x \mid x$ is an even positive integer$\}$ means $S = \{2, 4, 6, 8, ...\}$

- $n(A)$ means the number of elements in set A.

Definition: We say two sets are *equal* if they have exactly the same elements.

Subsets:

Definition: If each element of a set A is also an element of set B, we say that A is a *subset* of B. This is denoted $A \subseteq B$ or $A \subset B$. If A is not a subset of B, we write $A \not\subset B$.

$$A \subseteq B \quad \text{or} \quad A \subset B$$

Definition: We say A is a *proper subset* of B if $A \subseteq B$ but $A \neq B$. (In other words, every element of A is also an element of B, but B contains at least one element that is not in A.)

Note on notation: Some books use the symbol $\subset$ to indicate a proper subset. Some books use $\subset$ to indicate any subset, proper or not.

Definition: The set of all elements under consideration is called the *universal set*, usually denoted U.
    Example: If you're dealing with sets of real numbers, then U is the set of all real numbers. So "Wednesday" would not be an element of U, but 5.7 would be in U.

Example 1: Consider these sets.

$A = \{1, 2, 3, 4, 5, 6\}$

$B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$C = \{1, 3, 5, 2, 4, 6\}$

$A = C$

$A \subseteq B \quad (A \text{ is a subset of } B)$

Also $C \subseteq B$

Note:
- $\varnothing$ is a subset of every set. (i.e. $\varnothing \subseteq A$ for every set A.)
- Every set is a subset of itself. (i.e. $A \subseteq A$ for every set A.)

Example 2: List all subsets of $\{1, 2, 3\}$.

$\{1, 2\}, \quad \{1, 3\}, \quad \{2, 3\}, \quad \{1\}, \{2\}, \{3\}, \quad \varnothing, \quad \{1, 2, 3\}$

Note: If a set has n elements, how many subsets does it have?

2^n. So a set with 3 elements has $2^3 = 8$ subsets

Set operations:

- Union $\cup$: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

    $A \cup B$ Key Word: OR

- Intersection $\cap$: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

    $A \cap B$ Key Word: AND

- Complement A' or A^C or $A^\sim$: $A' = \{x \in U \mid x \notin A\}$.

    A', A^C, $A^\sim$ Key Word: NOT

Note: $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.

   $(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.

Definition: We say that A and B are *disjoint sets* if $A \cap B = \varnothing$.

Example 3: $U = \{1,2,3,4,5,6,7,8\}$

   $H = \{1,3,5,7\}$

   $K = \{1,2,3\}$

   $J = \{2,4,6,8\}$

   $L = \{1,2\}$

$H \cap K = \{1,3\}$

$J \cap K = \{2\}$

$H \cup K = \{1,3,5,7,2\} = \{1,2,3,5,7\}$

$L \cup J = \{1,2,4,6,8\}$

$H \cup J = \{1,2,3,4,5,6,7,8\} = U$

$K' = K^C = \{4,5,6,7,8\}$ $L^C = \{3,4,5,6,7,8\}$

Venn Diagrams: These help us visualize set relationships and operations.

Example 4: Draw Venn diagrams for $A \cup B$, $A \cap B$, A^C, B^C, $(A \cap B)^C$, and $(A \cup B)^C$.

AUB

A∩B

A^C

(A∩B)^C

(A∪B)^C

A^C∩B