1.5: Multiplication and Division of Real Numbers

We can indicate multiplication with a dot or with parentheses.

Ex: $3(5) = \boxed{15}$ or $3 \cdot 5 = \boxed{15}$

Ex: $7(-2) = \boxed{-14}$ (don't write $7 \cdot -2$)

Ex: $-6(-8) = \boxed{48}$

Ex: $\frac{3}{5} \cdot \frac{2}{7} = \frac{3(2)}{5(7)} = \boxed{\frac{6}{35}}$

Ex: $-\frac{1}{4}(-6) = -\frac{1}{4}\left(\frac{-6}{1}\right) = \frac{\cancel{6}^{3}}{\cancel{4}_{2}} = \frac{6 \div 2}{4 \div 2} = \boxed{\frac{3}{2}}$

Division

Ex: $\frac{40}{-8} = \boxed{-5}$ because $-8(-5) = 40$

Ex: $\frac{-40}{-8} = \boxed{5}$ because $-8(5) = -40$

Note: $\frac{40}{8}$ means the same as $40 \div 8$

Ex: $\frac{-2}{8} = \boxed{-\frac{1}{4}}$

Ex: $\frac{2}{-8} = \boxed{-\frac{1}{4}}$

Ex: $-\frac{2}{8} = \boxed{-\frac{1}{4}}$

Ex: $\frac{-2}{-8} = \boxed{\frac{1}{4}}$

So, all of these are equivalent (the same)

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$$

Important: Division by 0 is undefined. We can never have a zero denominator.

Ex: $\frac{0}{12} = \boxed{0}$

because $12(0) = 0$

Ex: $\frac{12}{0}$ is undefined

because $0(?) = 12$ is impossible

What about $\frac{0}{0}$?

$0(\quad) = 0$

you could put anything in here. So we don't get a unique answer.

So, $\frac{0}{0}$ is also undefined (sometimes called indeterminate)

Ex: $-\frac{3}{5}(-6)\left(-\frac{1}{4}\right)$

$= -\frac{3}{5}\left(\frac{-6}{1}\right)\left(-\frac{1}{4}\right)$

$= \frac{18}{5}\left(-\frac{1}{4}\right)$

$= -\frac{\cancel{18}^{9}}{\cancel{20}_{10}} = \boxed{-\frac{9}{10}}$

Division with fractions:

Ex: $\frac{2}{3} \div 4$

$= \frac{2}{3} \div \frac{4}{1}$

$= \frac{2}{3} \cdot \frac{1}{4}$

$= \frac{\cancel{2}^{1}}{\cancel{12}_{6}} = \boxed{\frac{1}{6}}$

Dividing by a number is equivalent to multiplying by its reciprocal.

Note: Reciprocal of 4 is $\frac{1}{4}$

Reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$

Reciprocal of $-\frac{2}{7}$ is $-\frac{7}{2}$.

Two numbers are reciprocals if their product is 1.

(Product is what you get when you multiply)

(sum is what you get when you add)

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Ex.: $$\frac{\frac{4}{3}}{\frac{2}{5}} = \frac{4}{3}\cdot\frac{5}{2} = \frac{\cancel{20}^{10}}{\cancel{6}_{3}} = \boxed{\frac{10}{3}}$$

1.6: Properties of Real Numbers and Simplifying Expressions

Properties of Real Numbers

Commutative Property of Addition: $a+b=b+a$

(commutative means we can change the order)

$2+3=3+2$
(both equal 5)

Commutative Property of Multiplication: $ab=ba$

Note: $ab = a\cdot b = a(b) = (a)(b)$
Do not use $\times$ for multiplication.

Ex: $5(3)=3(5)$
(both equal 15)

Division and subtraction are not commutative
(cannot change order)

$3-8 \neq 8-3$ $\qquad$ $\frac{12}{3} \neq \frac{3}{12}$

Associative Property of Addition: $(a+b)+c = a+(b+c)$

(associative means you can regroup)

Ex:

$2+3+4 = (2+3)+4 = 2+(3+4)$
(both equal 9)

Associative Property of Multiplication: $abc = (ab)c = a(bc)$

Ex: $2(3\cdot 4) = (2\cdot 3)(4)$
(both equal 24)

Distributive Property of Real Numbers:

(multiplication distributes over addition)

$$a(b+c) = ab + ac$$

$$(a+b)\cdot c = ac + bc$$

Ex: $2(3+4)$

$= 6 + 8$

$= \boxed{14}$

Ex: $2 + (3+4)$

$= 2 + 7 = \boxed{9}$

Algebraic Expression: Numbers and variables combined through arithmetic operations.

Note: an expression does not have an equals sign.

Examples of Expressions: $2x+5$ $\quad \sqrt{a+76}$

$4x^2 - 96x + 1$

$8x^2y$

Algebraic expressions being added are called Terms.

Algebraic expressions being multiplied are called factors.

The coefficient is the numerical factor in a term.

Ex: $3x^2 - 7x + 5$ has 3 terms.

The coefficient of the x^2 term is 3.

The coefficient of the x term is -7.

The constant (constant coefficient) is 5.

<u>Like terms</u> have the same variables and exponents.

We <u>combine like terms</u> by adding the coefficients.

<u>Ex</u>: $-7x^2$, $6x^2$, $\frac{3}{5}x^2$, x^2 are like terms

<u>Ex</u>: $-7x^2$, $8x$ are unlike terms.

<u>Ex</u>: Simplify.

$$2x^2 - 8x - 5 - 3x^2 - \frac{1}{2}x - 17$$

$$= -1x^2 - 8x - \frac{1}{2}x - 22$$

$$= -x^2 - \frac{8}{1}x\left(\frac{2}{2}\right) - \frac{1}{2}x - 22$$

$$= -x^2 - \frac{16}{2}x - \frac{1}{2}x - 22$$

$$= \boxed{-x^2 - \frac{17}{2}x - 22} \text{ same as } \boxed{-x^2 - \frac{17x}{2} - 22}$$

Why? $-\frac{17}{2}x = -\frac{17}{2}\cdot\frac{x}{1} = -\frac{17x}{2}$

Do <u>not</u> write $-\frac{17}{2x}$ or $-17/2x$

(it's not clear whether the x is meant to be in the denominator)

<u>Ex</u>: Simplify. $2(5-3x^2+7x) - 8(-5-6x+x^3)$

$$= 10 - 6x^2 + 14x + 40 + 48x - 8x^3$$

$$= \boxed{-8x^3 - 6x^2 + 62x + 50}$$

Ex: $4 - 5a^2 + 2a - (-a^2 - 6a + 4)$

$= 4 - 5a^2 + 2a - 1(-1a^2 - 6a + 4)$

$= 4 - 5a^2 + 2a + a^2 + 6a - 4$

$= -4a^2 + 8a + 0$

$= \boxed{-4a^2 + 8a}$

Ex: Evaluate $x - y^2$ for $x = 0$, $y = 5$.

substitute $x=0$, $y=5$: $0 - 5^2$

$= 0 - 25$

$= \boxed{-25}$

Ex: Evaluate $b^2 - 4ac$ for $a = -7$, $b = -2$, $c = 3$.

$(-2)^2 - 4(-7)(3)$

$= 4 + 28(3)$

$= 4 + 84$

$= \boxed{88}$

$$\begin{array}{r} \overset{2}{2}8 \\ \times 3 \\ \hline 84 \end{array}$$

Recall:

$-2^2 = -4$

$(-2)^2 = 4$

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Ex: Simplify.

$$\frac{2}{3}\left(4x - \frac{1}{4}\right) - \frac{3}{8}\left(\frac{1}{2}x - \frac{1}{3}\right)$$

$$= \frac{2}{3}(4x) + \frac{2}{3}\left(-\frac{1}{4}\right) - \frac{3}{8}\left(\frac{1}{2}x\right) - \frac{3}{8}\left(-\frac{1}{3}\right)$$

$$= \frac{8}{3}x - \frac{\cancel{2}^{1}}{\cancel{12}_{6}} - \frac{3}{16}x + \frac{\cancel{3}^{1}}{\cancel{24}_{8}}$$

$$= \frac{8}{3}x - \frac{1}{6} - \frac{3}{16}x + \frac{1}{8}$$

$$= \frac{8}{3}x\left(\frac{16}{16}\right) - \frac{3}{16}x\left(\frac{3}{3}\right) - \frac{1}{6}\left(\frac{4}{4}\right) + \frac{1}{8}\left(\frac{3}{3}\right)$$

$$= \frac{128}{48}x - \frac{9}{48}x - \frac{4}{24} + \frac{3}{24}$$

$$= \boxed{\frac{119}{48}x - \frac{1}{24}}$$