

2.1: Addition, Subtraction, Multiplication, and Division Properties of Equality

2.1.1

Note Title

9/18/2017

An equation is a mathematical statement claiming two values are equal.

An equation has an equals sign.

A solution for an equation is a value for the variable that makes the equation true.

To solve an equation means to find its solution(s).

Examples of equations:

$$2y - 5 = 11$$

$$6 = 2 \quad \leftarrow \text{this one is always false.}$$

Ex. Is 3 a solution of $2y - 5 = 11$?

Substitute $y = 3$:

$$2(3) - 5 = 11$$

$$6 - 5 = 11$$

$$1 = 11$$

False

No, 3 is not a solution

Is 8 a solution of $2y - 5 = 11$?

Substitute $y = 8$:

$$2(8) - 5 = 11$$

$$16 - 5 = 11$$

$$11 = 11 \quad \text{True}$$

Yes, 8 is a solution.

Addition (and subtraction) Property of Equality

$$\text{If } a = b, \text{ then } a + c = b + c$$

This means we can add (or subtract) the same quantity on both sides.

Example:

Solve.

$$x - 7 = 15$$

$$+7 \quad +7$$

$$x + 0 = 15 + 7$$

$$x = 15 + 7$$

$$x = 22$$

Solution:
Set

$$\{22\}$$

Solution set: Set of values that make the equation true.

Example:

Solve.

$$4 = y + 12$$

$$-12$$

$$-12$$

$$4 - 12 = y$$

$$-8 = y$$

Solution set:

$$\{-8\}$$

Example: Solve.

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$$\frac{5}{8} = a + \frac{1}{8}$$

$$\frac{5}{8} - \frac{1}{8} = a$$

$$a = \frac{5}{8} - \frac{1}{8}$$

$$a = \frac{4}{8}$$

$$a = \frac{1}{2}$$

Solution Set:

$$\left\{ \frac{1}{2} \right\}$$

Multiplication Property of Equality

If $a = b$, then $ac = bc$ for $c \neq 0$.

This means we can multiply (or divide) both sides by the same nonzero number.

This also gives us: If $a = b$, then $\frac{a}{c} = \frac{b}{c}$ for $c \neq 0$.

$\neq$ means "not equal to"

Ex: Solve.

$$9x = 45$$

$$\frac{9x}{9} = \frac{45}{9}$$

$$1x = 5$$

$$x = 5$$

Solution set:

$$\{5\}$$

Ex: Solve. $-8x = 32$

$$\frac{-8x}{-8} = \frac{32}{-8}$$

$$x = -4$$

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Solution Set: $\{-4\}$

Ex: Solve. $-\frac{2}{3}y = 7$

$$\frac{-\frac{2}{3}y}{-\frac{2}{3}} = \frac{7}{-\frac{2}{3}}$$

$$y = 7 \left(-\frac{3}{2} \right)$$

$$y = -\frac{21}{2}$$

Recall: Dividing by a number is equivalent to multiplying by its reciprocal.

Solution Set: $\left\{ -\frac{21}{2} \right\}$

or $-\frac{2}{3}y = 7$

$$\left(-\frac{3}{2} \right) \left(-\frac{2}{3}y \right) = \left(-\frac{3}{2} \right) (7)$$

$$+ \frac{6}{6}y = -\frac{21}{2}$$

$$y = -\frac{21}{2}$$

Solution set: $\left\{ -\frac{21}{2} \right\}$ (same as before)

Example: $4x + 1 = 1$

$$4x = 0$$

$$\frac{4x}{4} = \frac{0}{4}$$

$$x = 0$$

Check:

$$x \neq 0: 4x + 1 = 1$$

$$4(0) + 1 = 1$$

$$0 + 1 = 1$$

$$1 = 1 \checkmark$$

Solve.

Recall:

$$\frac{0}{4} = 0$$

$\frac{4}{0}$ is undefined.

Solution Set: $\{0\}$

Ex.

Solve,

$$0.35x = 8.25$$

$$\boxed{2.15}$$

$$\frac{0.35x}{0.35} = \frac{8.25}{0.35}$$

$$x = \frac{8.25}{0.35} \left(\frac{100}{100} \right)$$

$$x = \frac{825}{35}$$

$$x = \frac{825 \div 5}{35 \div 5}$$

$$x = \frac{165}{7}$$

Sol'n set:

$$\boxed{\left\{ \frac{165}{7} \right\}}$$

$$\begin{array}{r} 165 \\ 5 \overline{) 825} \\ \underline{5} \\ 32 \\ \underline{30} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

2.2: Solving Linear Equations

2.2.1

A linear equation has the variable to the 1st power only. It can be rearranged to get $ax + b = c$ (or similar).

(here, x is the variable and a, b, c are numbers)

Equivalent Equations have the same solution set.

When solving an equation, each line should be an equation that is equivalent to the original equation (and equivalent to all the other steps).

Reminder: Expressions get simplified.
Equations get solved.

Ex: Solve.

$$5a + 9 = 7a - 1$$

$+1$ $+1$

$$5a + 10 = 7a$$

$-5a$ $-5a$

$$10 = 7a - 5a$$

$$10 = 2a$$

$$\frac{10}{2} = \frac{2a}{2}$$

$$5 = a$$

Solution set: $\{5\}$

OR

$$5a + 9 = 7a - 1$$

$-7a$ $-7a$

$$5a - 7a + 9 = -1$$

$$-2a + 9 = -1$$

-9 -9

$$-2a = -1 - 9$$

$$-2a = -10$$

Ex. Solve.

$$-5 - 2(y-5) = 4y - 2(y+4) - 3$$

$$-5 - 2y + 10 = 4y - 2y - 8 - 3$$

$$-2y + 5 = 2y - 11$$

$$-4y + 5 = -11$$

$$-4y = -16$$

$$\frac{-4y}{-4} = \frac{-16}{-4}$$

$$y = 4$$

$$\text{Sol'n Set: } \{4\}$$

Example. Solve.

$$4x - \frac{3}{5} = \frac{1}{3}x + \frac{1}{2}$$

$$4x - \frac{1}{3}x - \frac{3}{5} = \frac{1}{2}$$

$$4x - \frac{1}{3}x = \frac{1}{2} + \frac{3}{5}$$

$$\left(\frac{3}{3}\right) \frac{4x}{1} - \frac{1}{3}x = \frac{1}{2} \left(\frac{5}{5}\right) + \frac{3}{5} \left(\frac{2}{2}\right)$$

$$\frac{12}{3}x - \frac{1}{3}x = \frac{5}{10} + \frac{6}{10}$$

$$\frac{11}{3}x = \frac{11}{10}$$

$$\left(\frac{3}{11}\right) \frac{11}{3}x = \frac{11}{10} \left(\frac{3}{11}\right)$$

$$x = \frac{33}{110}$$

$$x = \frac{33 \div 11}{110 \div 11}$$

$$x = \frac{3}{10}$$

$$\text{Sol'n Set: } \left\{\frac{3}{10}\right\}$$