2.2.3

2.2: Solving Linear Equations (cont'd)

Example: $4(7x-2) = 2(14x-1)$. Solve.

$$28x - 8 = 28x - 2$$
$$+8 \qquad\qquad +8$$

$$28x = 28x + 6$$
$$-28x \qquad -28x$$

$$0x = 6$$

$$0 = 6$$ False!

(false for every x)
There is no x that will make this true.

$$\boxed{\text{No Solution}}$$

Note: The solution set is $\emptyset$, which means "the empty set"

Example: Solve.

$$-2(3-x) + 5x = 7x - 6$$

$$-6 + 2x + 5x = 7x - 6$$

$$-6 + 7x = 7x - 6$$
$$+6 \qquad\qquad +6$$

or $-6+7x = 7x-6$
$$-7x \qquad -7x$$

$$-6 = -6$$
$$+6 \qquad +6$$

$$0 = 0$$

$$7x = 7x$$
$$-7x \qquad -7x$$

$0 = 0$ True (true for every x)

Solution set: $\boxed{\text{All real numbers}}$

2.2.4

Summary

* If the variable disappears and leaves you with a false statement (ex: $0=6$ or $1=4$), then the solution set is No Solution.

This type of equation is called a contradiction. (an equation that cannot be true)

* If the variable disappears and leaves you with a true statement (ex: $0=0$ or $6=6$), then the solution set is All Real Numbers.

This type of equation is called an identity. (an equation that is always true)

* If the variable doesn't disappear (if we can isolate the variable), the equation has one numerical solution. (ex: $x=5$ or $h=-2$)

solution set $\{5\}$ or $\{-2\}$

This type of equation is called a conditional equation. (equation that is true for some value(s) of the variable but false for all other values)

2.3: Clearing Fractions and Decimals

(an alternative method of handling fractions)

To clear fractions, multiply both sides by the least common denominator (LCD) of all the fractions.

Example: Solve.

$$\frac{1}{5}y + 3 = \frac{7}{10}$$

LCD: 10

multiply both sides by 10:

$$10\left(\frac{1}{5}y + 3\right) = \left(\frac{7}{10}\right)(10)$$

$$\frac{10}{1}\left(\frac{1}{5}y\right) + 10(3) = \frac{7}{10}\frac{(10)}{1}$$

$$\frac{\cancel{10}^{2}}{\cancel{5}_{1}}y + 30 = \frac{\cancel{70}^{7}}{\cancel{10}_{1}}$$

$$\begin{aligned} 2y + 30 &= 7 \\ -30 \quad & \quad -30 \end{aligned}$$

$$2y = -23$$

$$\frac{2y}{2} = \frac{-23}{2}$$

$$y = -\frac{23}{2}$$

Sol'n Set: $\boxed{\left\{-\frac{23}{2}\right\}}$

(This is a conditional equation)

Example: Solve.

$$\frac{3}{4}x - \frac{2}{3} + \frac{1}{6} = 4x - \frac{5}{2}$$

LCD: 12

multiply both sides by 12:

$$\frac{(\cancel{12}^{3})}{1}\frac{3}{\cancel{4}_{1}}x - \frac{2}{\cancel{3}_{1}}\frac{(\cancel{12}^{4})}{1} + \frac{1}{\cancel{6}_{1}}\frac{(\cancel{12}^{2})}{1} = 4x(12) - \frac{5}{\cancel{2}_{1}}\frac{(\cancel{12}^{6})}{1}$$

$$\frac{36}{4}x - \frac{24}{3} + \frac{12}{6} = 48x - \frac{60}{2}$$

$$9x - 8 + 2 = 48x - 30$$

$$\underset{-9x}{9x} - 6 = \underset{-9x}{48x} - 30$$

$$\underset{+30}{-6} = 39x \underset{+30}{-30}$$

$$24 = 39x$$

$$\frac{\cancel{24}^{8}}{\cancel{39}_{13}} = \frac{39x}{39}$$

$$\frac{8}{13} = x$$

$$\begin{array}{r} 13 \\ 3\overline{)39} \\ \underline{3} \\ 09 \\ \underline{9} \\ 0 \end{array}$$

Sol'n Set: $\boxed{\left\{\frac{8}{13}\right\}}$

$\boxed{2.3.3}$

To clear decimals, multiply both sides by a power of 10.

Example: Solve.

$$3(-0.9n+0.5) = -3.5n+1.3$$

$$-2.7n+1.5 = -3.5n+1.3$$

multiply both sides by 10:

$$-2.7n(10)+1.5(10) = -3.5n(10)+1.3(10)$$

$$\begin{aligned} -27n+15 &= -35n+13 \\ \underset{+35n}{}\ \underset{-15}{} &\quad \underset{+35n}{}\ \underset{-15}{} \\ 8n &= -2 \end{aligned}$$

$$\frac{8n}{8} = \frac{-2}{8}$$

$$n = -\frac{1}{4}$$

Sol'n set: $\boxed{\left\{-\frac{1}{4}\right\}}$

Ex: Solve

$$0.75(m-2)+0.25m = 0.5$$

multiply both side by 100:

$$100(0.75)(m-2)+0.25m(100) = 0.50(100)$$

$$75(m-2)+25m = 50$$

$$75m-150+25m = 50$$

$$\begin{aligned} 100m-150 &= 50 \\ +150 &\quad +150 \end{aligned}$$

$$100m = 200$$

$$\frac{100m}{100} = \frac{200}{100}$$

$$m = 2$$

Sol'n Set: $\boxed{\{2\}}$

2.8: Linear Inequalities

Recall:

$<$ means "is less than"

$>$ means "is greater than"

$\leq$ means "is less than or equal to"

$\geq$ means "is greater than or equal to"

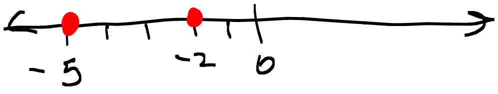
Note: $\leqslant$ and $\leq$ are equivalent

$\geqslant$ and $\geq$ are equivalent.

True/false:

$2 < 5$ True

$-5 > -2$ False



$2 < 2$ False

$2 \leq 2$ True

$-1 < -4$ False

$6 > -12$ True

$2 \leq 7$ True

Def'n: A linear inequality (in x) is any relationship that can be rearranged to one of these forms:

$ax+b<c$ $\quad$ $ax+b>c$

$ax+b\leq c$ $\quad$ $ax+b\geqslant c$

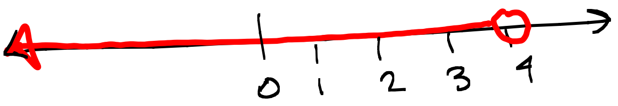
Examples: $5x-3<8$

$2x-7 \leq 5(16-x)$

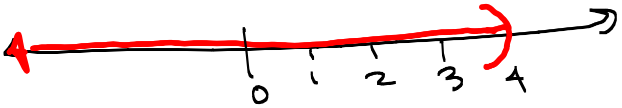
2.8.2

We can graph the solution set on a number line.

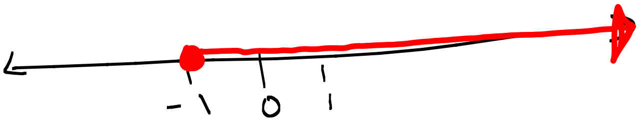
$x < 4$



OR



$x \geq -1$



or

Set-builder (set roster) notation:

Inequality:

$x \leq 4$

$\{x \mid x \leq 4\}$

← "such that"

Read: " The set of all x such that $x \leq 4$ "

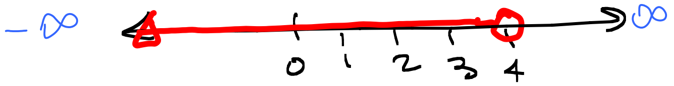
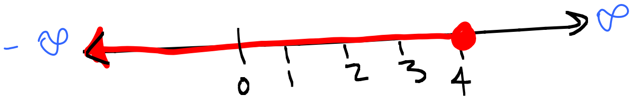
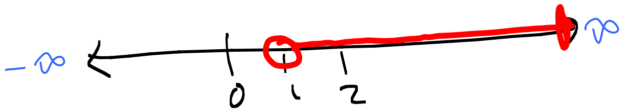
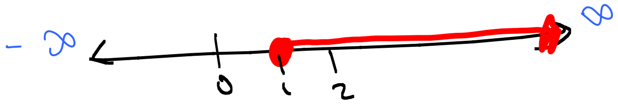
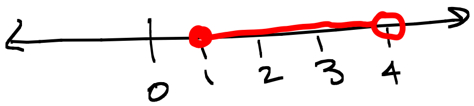
Interval notation:

* We use a comma to separate left and right boundaries of shaded area

* we use [to indicate boundary point is included
(to indicate boundary point is not included

* we use ∞ to represent right end of number line
$-\infty$ to represent left end of number line

2.8.3

Set-builder notation	Graph	Interval notation
$\{x \mid x < 4\}$		$(-\infty, 4)$
$\{x \mid x \leq 4\}$		$(-\infty, 4]$
$\{x \mid x > 1\}$		$(1, \infty)$
$\{x \mid x \geq 1\}$		$[1, \infty)$
$\{x \mid 1 \leq x < 4\}$		$[1, 4)$

To solve linear inequalities, we can

* add or subtract the same quantity on both sides
* multiply both sides (or divide) by the same positive number.
* we can multiply (or divide) both sides by the same negative number if we reverse the inequality sign

Ex: $2x + 1 < 7$

$$-1 \quad -1$$

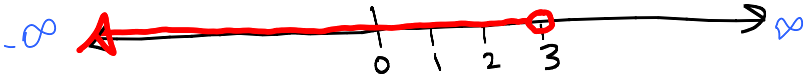
$$2x < 6$$

$$\frac{2x}{2} < \frac{6}{2}$$

$$x < 3$$

Set Builder notation: $\boxed{\{x \mid x < 3\}}$

Interval notation: $\boxed{(-\infty, 3)}$



$\boxed{2.8.4}$

Ex: Solve.

$$-3x < 12$$

reverse the inequality sign →

$$\frac{-3x}{-3} > \frac{12}{-3}$$

$$x > -4$$

Set builder notation: $\boxed{\{x \mid x > -4\}}$
(set roster)

Interval notation: $\boxed{(-4, \infty)}$

Note: If you have $-4 < x$, you can rearrange to get $x > -4$.
(if the variable is on the left, it is much easier to shade the number line correctly).