Note Title 10/4/2017

Example: (from RR8)

Solve. $-3 \le 1-2x < 9$

$-3 \le 1-2x$ **and** $1-2x < 9$

$-1 \quad -1 \qquad\qquad -1 \quad -1$

$-4 \le -2x \qquad\qquad -2x < 8$

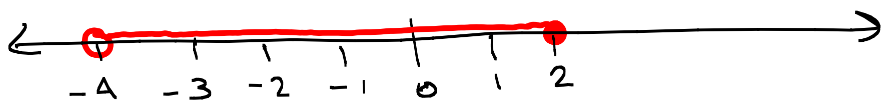
Reverse the inequality sign $\rightarrow$ $\dfrac{-4}{-2} \ge \dfrac{-2x}{-2} \qquad\qquad \dfrac{-2x}{-2} > \dfrac{8}{-2}$ [Reverse the inequality sign]

$2 \ge x$ **and** $x > -4$

Combine them: $2 \ge x > -4$

Rewrite smallest to largest: $-4 < x \le 2$

Set-builder notation: $\boxed{\{x \mid -4 < x \le 2\}}$



Interval notation: $\boxed{(-4, 2]}$

2.6: More applications of linear equations

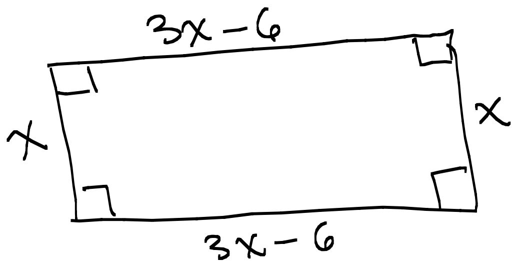
Ex: The perimeter of a rectangle is 100 ft. The length is 6 ft less than $\overbrace{\text{three times the } \underbrace{\text{width}}_{x}}^{3x}$. Find the length and width.

length: $3x - 6$

width: x

length $\xrightarrow[\text{to}]{\text{compare}}$ width

x



Perimeter = 100

$x + (3x-6) + x + (3x-6) = 100$

$x + 3x - 6 + x + 3x - 6 = 100$

$8x - 12 = 100$

$8x = 112$

$\dfrac{8x}{8} = \dfrac{112}{8}$

$x = 14$

OR

2(length) + 2(width) = Perimeter

$2(3x-6) + 2(x) = 100$

$6x - 12 + 2x = 100$

same

$8x - 12 = 100$

$$\begin{array}{r} 14 \\ 8\overline{)112} \\ \underline{8} \\ 32 \end{array}$$

width: $x = 14$

length: $3x - 6$

$x = 14 \Rightarrow 3(14) - 6$

$= 42 - 6$

$= 36$

$\boxed{\text{The width is 14 ft and the length is 36 ft.}}$

5.1: Multiplying and Dividing Expressions with Common Bases

5.1.1

Know these powers:

$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$2^{10} = 1024$

$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$

$4^2 = 16$
$4^3 = 64$
$4^4 = 256$

$5^2 = 25$
$5^3 = 125$
$5^4 = 625$

$6^2 = 36$
$6^3 = 216$

$7^2 = 49$
$7^3 = 343$

$8^2 = 64$
$8^3 = 512$

$9^2 = 81$
$9^3 = 729$

$10^2 = 100$
$10^3 = 1000$
$10^4 = 10000$
$\vdots$
$10^7 = 10\ 000\ 000$

Powers of positive and negative numbers

Example:

$$(-3)^2 = (-3)(-3) = \boxed{9}$$

$$-3^2 = -(3)(3) = \boxed{-9}$$

Ex:

$$(-2)^6 = \underbrace{(-2)(-2)}_{+}\underbrace{(-2)(-2)}_{+}\underbrace{(-2)(-2)}_{+} = \boxed{64}$$

$$(-2)^5 = \underbrace{(-2)(-2)}_{+}\underbrace{(-2)(-2)}_{+}\underbrace{(-2)}_{-} = \boxed{-32}$$

$$-2^6 = -(2)(2)(2)(2)(2)(2) = \boxed{-64}$$

$$-2^5 = -(2)(2)(2)(2)(2) = \boxed{-32}$$

equal

Important:

* If you raise a negative number to an <u>even</u> power, you get a <u>positive</u> result.

* If you raise a negative number to an <u>odd</u> power, you get a <u>negative</u> result.

<u>Ex:</u> $(-3)^4 = \boxed{81}$

$(-3)^5 = \boxed{-243}$

<u>Ex:</u> $-\underbrace{(-1)^{11}} = -(\underbrace{-1}) = \boxed{1}$

$-\underbrace{(-1)^{12}} \quad -\underbrace{1} = \boxed{-1}$

Properties (Laws) of Exponents

1) $x^a x^b = x^{a+b}$ <u>Ex:</u> $x^2x^3 = (xx)(xxx) = x^5$

2) $(x^a)^b = x^{ab}$ <u>Ex:</u> $(x^2)^3 = (xx)^3 = (xx)(xx)(xx) = x^6$

3) $\dfrac{x^a}{x^b} = x^{a-b}$ <u>Ex:</u> $\dfrac{x^6}{x^4} = \dfrac{xx\cancel{x}\cancel{x}\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}\cancel{x}\cancel{x}} = \dfrac{xx}{1} = xx = x^2$

4) $(xy)^a = x^a y^a$ <u>Ex:</u> $(xy)^3 = (xy)(xy)(xy) = x^3y^3$

5) $\left(\dfrac{x}{y}\right)^a = \dfrac{x^a}{y^a}$ <u>Ex:</u> $\left(\dfrac{x}{y}\right)^3 = \left(\dfrac{x}{y}\right)\left(\dfrac{x}{y}\right)\left(\dfrac{x}{y}\right) = \dfrac{xxx}{yyy} = \dfrac{x^3}{y^3}$

Ex: $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \boxed{\frac{16}{81}}$ ← Prop. 5

Ex: $\left(-\frac{1}{5}\right)^3 = -\frac{1^3}{5^3} = \boxed{-\frac{1}{125}}$

Ex: $\frac{3^{11}}{3^9} = 3^{11-9} = 3^2 = \boxed{9}$

Ex: $2x^4x^5 = 2x^{4+5} = \boxed{2x^9}$ ← Prop. #1

Ex: $2x^4 + x^5$ cannot be simplified further (no like terms)

Ex: $(6a^4)(-3a^2) = -18a^{4+2} = \boxed{-18a^6}$

Ex: $a^3 \cdot a \cdot a^5 = a^{3+1+5} = \boxed{a^9}$

Ex: $\frac{g^6}{g^4} = g^{6-4} = \boxed{g^2}$

Ex: $\frac{y^3y^6}{y^4} = \frac{y^9}{y^4} = y^{9-4} = \boxed{y^5}$

Ex: $(-2c^2d^3)(-6c^3d) = \boxed{12c^5d^4}$

Ex:

$$\frac{\overset{5}{\cancel{15}}\, m^5\, n^{12}}{\underset{1}{\cancel{3}}\, m^4\, n^2} = \frac{5mn^{10}}{1} = \boxed{5mn^{10}}$$

(5.1.4)

5.2: More properties of exponents

Recall
Prop #2: $(x^a)^b = x^{ab}$

simplify

Ex: $(y^3)^5 = y^{3(5)} = \boxed{y^{15}}$

Ex: $(3xy)^4 = 3^4 x^4 y^4 = \boxed{81x^4y^4}$

Prop. #4

Ex: $(-2a)^4 = (-2)^4 a^4 = \boxed{16a^4}$

Ex: $(-2a)^5 = (-2)^5 a^5 = \boxed{-32a^5}$

Ex: $(x^2y^4)^3 = (x^2)^3(y^4)^3 = \boxed{x^6y^{12}}$

Prop #4

Prop. #2

Ex: $(3a^4b^2c)^5 = 3^5 a^{20} b^{10} c^5 = \boxed{243a^{20}b^{10}c^5}$

Prop #2 and #4

Ex: $(-f)^6 = (-1f)^6 = (-1)^6 f^6 = 1f^6 = \boxed{f^6}$

Ex: $(-3x^4y^2)^2(-2x^5yz)^3$

$= (-3)^2 x^8 y^4 (-2)^3 x^{15} y^3 z^3$

$= 9x^8y^4(-8)x^{15}y^3z^3 = \boxed{-72x^{23}y^7z^3}$

<u>Ex:</u> $$\frac{(2x^2yz)(-x^4y^2z^3)^4}{(3xy)^2}$$

$$= \frac{2x^2yz(-1)^4x^{16}y^8z^{12}}{3^2x^2y^2} = \frac{2(1)x^{18}y^9z^{13}}{9x^2y^2}$$

$$= \boxed{\frac{2x^{16}y^7z^{13}}{9}} = \boxed{\frac{2}{9}x^{16}y^7z^{13}}$$