5.3: Definitions of b^0 and b^{-n} 5.3.1

Note Title (Division with exponents and negative exponents) 10/9/2017

$\boxed{\text{Definition of a negative exponent: } \quad x^{-n} = \frac{1}{x^n}}$

Note: $x^{-n} = \frac{x^{-n}}{1} = \frac{1}{x^n}$

Also: $\frac{1}{x^{-n}} = \frac{x^n}{1}$ (because $\frac{1}{x^{-n}} = \frac{1}{\frac{1}{x^n}} = 1 \cdot \frac{x^n}{1}$

$= \frac{1}{1} \cdot \frac{x^n}{1} = x^n$)

Also $x^{-1} = \frac{x^{-1}}{1} = \frac{1}{x^1} = \frac{1}{x}$ (so x^{-1} is the reciprocal of x)

Simplify

Ex: $3^{-2} = \frac{3^{-2}}{1} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$

Ex: $-2^{-4} = -\frac{2^{-4}}{1} = -\frac{1}{2^4} = \boxed{-\frac{1}{16}}$

Ex: $(-3)^{-4} = \frac{(-3)^{-4}}{1} = \frac{1}{(-3)^4} = \boxed{\frac{1}{81}}$

Ex: $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$

OR $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$

Ex: $\frac{1}{4^{-2}} = \frac{4^2}{1} = \frac{16}{1} = \boxed{16}$

Ex: $\frac{3}{5^{-2}} = \frac{3 \cdot 5^2}{1} = \frac{3 \cdot 25}{1} = \frac{75}{1} = \boxed{75}$

Ex: $\left(-\frac{2}{3}\right)^{-4} = \left(\frac{-2}{3}\right)^{-4}$

$= \frac{(-2)^{-4}}{3^{-4}} = \frac{3^4}{(-2)^4}$

$= \boxed{\frac{81}{16}}$

Recall:

equal $\left\{ \begin{array}{l} -\frac{2}{4} = -\frac{1}{2} \\ \frac{-2}{4} = -\frac{1}{2} \\ \frac{2}{-4} = -\frac{1}{2} \end{array} \right.$

In this class, when simplifying expressions with variables, we will write all answers with positive exponents only.

simplify

Ex: $d^{-6} = \frac{d^{-6}}{1} = \boxed{\frac{1}{d^6}}$

Ex: $a^{-7} = \frac{a^{-7}}{1} = \boxed{\frac{1}{a^7}}$

Ex: $\frac{1}{n^{-8}} = \frac{n^8}{1} = \boxed{n^8}$

Ex: $2x^{-5} = \frac{2x^{-5}}{1} = \boxed{\frac{2}{x^5}}$

Note: $\frac{2x^{-5}}{1} = \frac{2}{1 \cdot x^5} = \frac{2}{x^5}$

Ex: $(2x)^{-5} = \frac{(2x)^{-5}}{1} = \frac{2^{-5}x^{-5}}{1} = \frac{1}{2^5 x^5} = \boxed{\frac{1}{32x^5}}$

★ **Important Fact:** ★

$x^0 = 1$, as long as $x \neq 0$.

Note: 0^0 is undefined.

Here's why: $\frac{2^3}{2^3} = \frac{8}{8} = 1$

must be equal

From the properties of exponents,

$\frac{2^3}{2^3} = 2^{3-3} = 2^0$

Recall

$x^a x^b = x^{a+b}$

$(x^a)^b = x^{ab}$

$\frac{x^a}{x^b} = x^{a-b}$

Also: $2^5 = 32$

$2^4 = 32 \div 2 = 16$

$2^3 = 16 \div 2 = 8$

$2^2 = 8 \div 2 = 4$

$2^1 = 4 \div 2 = 2$

$2^0 = 2 \div 2 = 1$ So $2^0 = 1$.

Example: $5^0 = \boxed{1}$

Ex: $(-7)^0 = \boxed{1}$

Ex: $(6x^4y^7z)^0 = \boxed{1}$ as long as $x \neq 0,\ y \neq 0,\ z \neq 0$

Ex: $a^{-5}a^3 = \dfrac{a^{-5}a^3}{1} = \dfrac{a^3}{a^5} = \dfrac{\cancel{a}\cancel{a}\cancel{a}}{\cancel{a}\cancel{a}\cancel{a}aa} = \boxed{\dfrac{1}{a^2}}$

Ex: $a^5a^{-3} = \dfrac{a^5a^{-3}}{1} = \dfrac{a^5}{a^3} = \dfrac{a^2}{1} = \boxed{a^2}$

Ex: $\dfrac{t^2}{t^{-8}} = \dfrac{t^2t^8}{1} = \dfrac{t^{10}}{1} = \boxed{t^{10}}$

Note: $\dfrac{t^2}{t^{-8}} = t^{2-(-8)} = t^{2+8} = \boxed{t^{10}}$

Ex: $\left(\dfrac{2x}{3}\right)^{-4} = \dfrac{2^{-4}x^{-4}}{3^{-4}} = \dfrac{3^4}{2^4x^4} = \boxed{\dfrac{81}{16x^4}}$

Ex: $(-3a^3b^2)^{-3} = \dfrac{(-3a^3b^2)^{-3}}{1} = \dfrac{(-3)^{-3}a^{-9}b^{-6}}{1}$

$= \dfrac{1}{(-3)^3a^9b^6} = \boxed{\dfrac{1}{-27a^9b^6}} = \boxed{-\dfrac{1}{27a^9b^6}}$

5.3.5

Ex:

$$\frac{x^{-3}y^{4}z}{x^{-8}y^{9}z^{-2}} = \frac{y^{4}z\,x^{8}\,z^{2}}{x^{3}\,y^{9}}$$

$$= \frac{x^{8}y^{4}z^{3}}{x^{3}y^{9}} = \boxed{\frac{x^{5}z^{3}}{y^{5}}}$$

Example:

$$\frac{-3x^{-5}\left(x^{-1}y^{3}z^{-2}\right)^{-4}}{\left(3x^{2}y^{4}\right)^{-2}\left(2xy^{-3}z\right)^{4}}$$

$$= \frac{-3x^{-5}\,x^{4}\,y^{-12}\,z^{8}}{3^{-2}x^{-4}y^{-8}\cdot 2^{4}x^{4}y^{-12}z^{4}}$$

$$= \frac{-3\,x^{4}\,z^{8}\cdot 3^{2}x^{4}y^{8}\,y^{12}}{x^{5}\,y^{12}\cdot 2^{4}x^{4}\,z^{4}}$$

$$= \frac{-3\cdot 9x^{8}y^{20}z^{8}}{16x^{9}y^{12}z^{4}} = \boxed{\frac{-27y^{8}z^{4}}{16x}}$$

We will come back to 5.4 if time allows.

5.5: Addition and subtraction of polynomials

A **polynomial** in one variable (x) is defined to be a sum of terms of the form ax^n, where a is a real number and n is a **nonnegative integer**.

Polynomials: $4x^3 - 6x^7 + 2x$

$\frac{1}{2}x^4 - \frac{1}{2}x^3 + 5 + x$

Not polynomials: $\frac{1}{x^5} + \frac{4}{x^3}$ ← same as $x^{-5} + 4x^{-3}$ (negative exponents)

Leading term: term with biggest exponent

Leading coefficient: coefficient of leading term.

Degree: highest exponent on a term

Ex: $-2x^3 - 4x^7 - 3x^9 + 5x$

Leading term: $-3x^9$

Leading coefficient: -3

Degree: 9