

# Review problems (taken from R210)

$$\begin{aligned}
 \textcircled{b} \quad (-x^2y^{-1})^{-3} &= \frac{(-x^2y^{-1})^{-3}}{1} = \frac{(-1)^{-3} x^{-6} y^3}{1} \\
 &= \frac{y^3}{(-1)^3 x^6} = \frac{y^3}{-1x^6} = \boxed{\frac{y^3}{-x^6}} \\
 &= \boxed{-\frac{y^3}{x^6}}
 \end{aligned}$$

$$\textcircled{c} \quad \frac{2}{3x^{-6}y^4} = \boxed{\frac{2x^6}{3y^4}}$$

$$\textcircled{d} \quad -3x^{-4} = \frac{-3x^{-4}}{1} = \boxed{\frac{-3}{x^4}} = \boxed{-\frac{3}{x^4}}$$

$$\textcircled{e} \quad 5^0 = \boxed{1}$$

# 5.5: Addition & subtraction of Polynomials (cont'd)

5.5.2

Ex: what is the degree of  $-2x^4 + 5x^2 - 3x^6 - 8x + 13$  ?

Leading term:  $-3x^6$   
 (term with the biggest exponent)

Leading coefficient:  $-3$

Degree: 6 (largest exponent)

Arrange in descending order:

$$-3x^6 - 2x^4 + 5x^2 - 8x + 13$$

## Special terms for certain polynomials

Polynomial with 3 terms: trinomial

Poly. with 2 terms: binomial

Poly. with 1 term: monomial

Give an example of a

(a) 4<sup>th</sup> degree binomial

Possible answers:  $-2x^3 + 16x^4$   
correct.

$6 - y^4$   
correct

$x^4 + x^2$   
correct

$3x^4 + 5x^2$   
Correct.

(b) 5<sup>th</sup> degree trinomial

Possible answers:  $15x^5 + x^3 + 6$   
correct

$3x^5 + 5x^2 - 1x$   
Correct

$4p - 3p^5 - 8p^3$   
Correct

(c) 2<sup>nd</sup> degree monomial

Possible answers:  $x^2$

$-3x^2$

$2x^2$

All ok.

$36p^2$

For a polynomial with 2 or more variables, the degree is the largest "total exponent" on a term. 5.5.3

Ex:  $-2xy^2 + 4x^2y^2 - 5xy^3 + 7x^2y^4 - 7xz^2 + z^3$

total exponent  $1+2=3$  (pointing to  $-2xy^2$ )  
 total exponent  $2+2=4$  (pointing to  $4x^2y^2$ )  
 total exponent  $1+3=4$  (pointing to  $-5xy^3$ )  
 total exponent  $2+4=6$  (pointing to  $7x^2y^4$ )  
 total exp  $1+2=3$  (pointing to  $-7xz^2$ )  
 total exp  $3$  (pointing to  $z^3$ )

leading term

Degree: 6

Adding and subtracting: we already know how to do this.

Simplify

Ex:  $(3x^2 - 7x + 5) + (-4x^2 + 8x + 1)$

$$= \underline{3x^2} - \underline{7x} + \underline{5} - \underline{4x^2} + \underline{8x} + \underline{1}$$

$$= \boxed{-x^2 + x + 6}$$

Ex:  $(2x^3 - 5x^2) - (-x^2 + 5x - 8)$

$$= 2x^3 - 5x^2 - 1(-x^2 + 5x - 8)$$

$$= 2x^3 - 5x^2 + x^2 - 5x + 8$$

$$= \boxed{2x^3 - 4x^2 - 5x + 8}$$

# 5.6: Multiplication of Polynomials and Special Products

5.6.1

## Multiplying Monomials

Ex.  $3x^4 (-7x^8)$   
 $= \boxed{-21x^{12}}$

Multiply a monomial times a polynomial with 2 or more terms:

we distribute.

Ex.  $3x^3 (2x^2 - 8x + 7)$

$= 3x^3 (2x^2) + 3x^3 (-8x) + 3x^3 (7)$

$= \boxed{6x^5 - 24x^4 + 21x^3}$

Ex.  $-5x^2y^3 (-xy^4 + 8xy - 3x^3y^5 - 2y^2)$

$= -5x^2y^3 (-1xy^4) - 5x^2y^3 (8xy) - 5x^2y^3 (-3x^3y^5) - 5x^2y^3 (-2y^2)$

$= \boxed{5x^3y^7 - 40x^3y^4 + 15x^5y^8 + 10x^2y^5}$

# Multiplying 2 polynomials with 2 or more terms. 5.6.2

Recall: distributive property:

$$c(a+b) = ca + cb = ac + bc$$

$$(a+b)c = ac + bc$$

Ex:  
multiply.

$$\begin{aligned} & (3x+5)(x-8) \\ &= 3x(x-8) + 5(x-8) \\ &= 3x^2 - 24x + 5x - 40 \\ &= \boxed{3x^2 - 19x - 40} \end{aligned}$$

Ex:

$$\begin{aligned} & (2x-y)(6x-5) \\ &= 2x(6x-5) - y(6x-5) \\ &= \boxed{12x^2 - 10x - 6xy + 5y} \end{aligned} \quad \text{(no like terms)}$$

$$\begin{aligned} \text{Ex: } & (-2x+8)(x^2+7x-4) \\ &= -2x(x^2+7x-4) + 8(x^2+7x-4) \\ &= -2x^3 - 14x^2 + 8x + 8x^2 + 56x - 32 \\ &= \boxed{-2x^3 - 6x^2 + 64x - 32} \end{aligned}$$

$$\underline{\text{Ex.}}: (x^2 - 7x - 1)(2x^2 - 5x - 6)$$

5.6.3

$$= x^2(2x^2 - 5x - 6) - 7x(2x^2 - 5x - 6) - 1(2x^2 - 5x - 6)$$

$$= 2x^4 - 5x^3 - 6x^2$$

$$- 14x^3 + 35x^2 + 42x$$

$$- 2x^2 + 5x + 6$$

$$= \boxed{2x^4 - 19x^3 + 27x^2 + 47x + 6}$$

The "FOIL" Method (for multiplying two binomials)

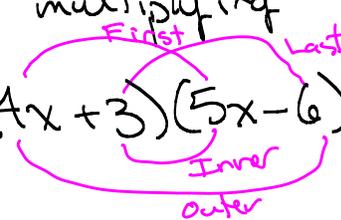
F: First

O: Outer

I: Inner

L: Last

Example:  $(4x + 3)(5x - 6)$



$$= 20x^2 - 24x + 15x - 18$$

$$= \boxed{20x^2 - 9x - 18}$$

Ex:

$$(x - 7)(2x - 5)$$

$$= 2x^2 - 5x - 14x + 35$$

$$= \boxed{2x^2 - 19x + 35}$$

5.6.4

## Difference of 2 Squares Pattern

$$(a+b)(a-b) = a^2 - b^2$$

Note:  $(a+b)(a-b) = a^2 - \cancel{ba} + \cancel{ba} - b^2 = a^2 - b^2$

Ex:  $(x+5)(x-5)$   
 $= x^2 - \cancel{5x} + \cancel{5x} - 25$   
 $= \boxed{x^2 - 25}$

## Perfect Squares

Example:

$$(x+6)^2$$
$$= (x+6)(x+6)$$
$$= x^2 + 6x + 6x + 36$$
$$= \boxed{x^2 + 12x + 36}$$

## 5.7: Division of Polynomials

5.7.1

### Dividing a polynomial by a monomial

Ex. Divide.

$$\frac{6x^4 - 8x^3 + 12x^2 - 9x}{2x^3}$$

$$= \frac{\overset{3}{\cancel{6}x^4}}{\overset{2}{\cancel{2}x^3}} - \frac{\overset{4}{\cancel{8}x^3}}{\overset{2}{\cancel{2}x^3}} + \frac{\overset{6}{\cancel{12}x^2}}{\overset{2}{\cancel{2}x^3}} - \frac{9x}{2x^3}$$

$$= \frac{3x}{1} - \frac{4}{1} + \frac{6}{x} - \frac{9}{2x^2}$$

$$= \boxed{3x - 4 + \frac{6}{x} - \frac{9}{2x^2}}$$

break into  
separate  
fractions

Note:

$$\begin{aligned} \frac{2}{7} + \frac{3}{7} \\ = \frac{2+3}{7} \\ = \frac{5}{7} \end{aligned}$$

### Dividing a polynomial by a polynomial

(when divisor has 2 or more terms)

Ex. Divide  $\frac{3x^2 + 7x + 9}{x + 2}$

We will need to do polynomial long division

$$x+2 \overline{) 3x^2 + 7x + 9}$$

Review:

5.7.2

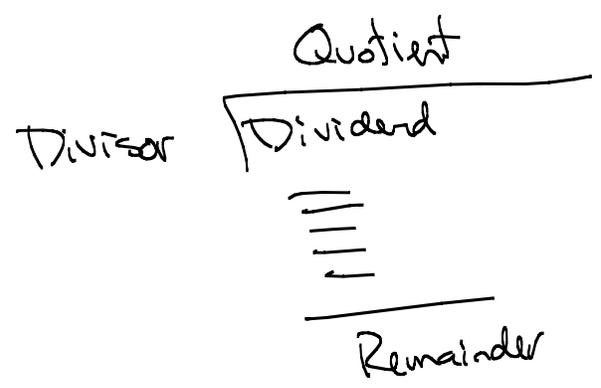
Numerical example:

Divide  $\frac{379}{12}$ .

$$\begin{array}{r} 31 \\ 12 \overline{) 379} \\ \underline{- 36} \phantom{0} \\ 19 \\ \underline{- 12} \\ 7 \end{array}$$

So  $\frac{379}{12} = 31 + \frac{7}{12}$

$(31)(12) + 7 = 379$



$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$

$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$