Review problems (taken from RR 10)

(b) $(-x^2y^{-1})^{-3} = \dfrac{(-1x^2y^{-1})^{-3}}{1} = \dfrac{(-1)^{-3}x^{-6}y^{3}}{1}$

$= \dfrac{y^3}{(-1)^3x^6} = \dfrac{y^3}{-1x^6} = \boxed{\dfrac{y^3}{-x^6}}$

$= \boxed{-\dfrac{y^3}{x^6}}$

(c) $\dfrac{2}{3x^{-6}y^4} = \boxed{\dfrac{2x^6}{3y^4}}$

(d) $-3x^{-4} = \dfrac{-3x^{-4}}{1} = \boxed{\dfrac{-3}{x^4}} = \boxed{-\dfrac{3}{x^4}}$

(e) $5^0 = \boxed{1}$

5.5: Addition & subtraction of Polynomials (cont'd)

Ex: what is the degree of

$$-2x^4 + 5x^2 - 3x^6 - 8x + 13 \; ?$$

Leading term: $-3x^6$
(term with the biggest exponent)

Leading coefficient: -3

$\boxed{\text{Degree: } 6}$ (largest exponent)

Arrange in descending order:

$$-3x^6 - 2x^4 + 5x^2 - 8x + 13$$

Special terms for certain polynomials

Polynomial with 3 terms: trinomial

Poly. with 2 terms: binomial

Poly. with 1 terms: monomial

Give an example of a

(a) 4th degree binomial

Possible answers: $-2x^3 + 16x^4$ correct. $6 - y^4$ correct $x^4 + x^2$ correct $3x^4 + 5x^2$ Correct.

(b) 5th degree trinomial

Possible answers: $15x^5 + x^3 + 6$ correct $3x^5 + 5x^2 - 1x$ Correct $4p - 3p^5 - 8p^3$ Correct

All OK.

(c) 2nd degree monomial

Possible answers: x^2 $-3x^2$ $2x^2$ $36p^2$

For a polynomial with 2 or more variables, the degree is the largest "total exponent" on a term.

leading term

Ex: $-2xy^2 + 4x^2y^2 - 5xy^3 + 7x^2y^4 - 7xz^2 + z^3$

total exponent $1+2=3$

total exponent $2+2=4$

total exponent $1+3=4$

total exponent $2+4=6$

total exp $1+2=3$

total exp 3

$$\boxed{\text{Degree: } 6}$$

Adding and subtracting: we already know how to do this.

Simplify

Ex: $(3x^2 - 7x + 5) + (-4x^2 + 8x + 1)$

$= 3x^2 - 7x + 5 - 4x^2 + 8x + 1$

$= \boxed{-x^2 + x + 6}$

Ex: $(2x^3 - 5x^2) - (-x^2 + 5x - 8)$

$= 2x^3 - 5x^2 - 1(-x^2 + 5x - 8)$

$= 2x^3 - 5x^2 + x^2 - 5x + 8$

$= \boxed{2x^3 - 4x^2 - 5x + 8}$

5.6: Multiplication of Polynomials and Special Products

Multiplying Monomials

Ex: $3x^4(-7x^8)$

$= \boxed{-21x^{12}}$

Multiply a monomial times a polynomial with 2 or more terms:

we distribute.

Ex: $3x^3(2x^2 - 8x + 7)$

$= 3x^3(2x^2) + 3x^3(-8x) + 3x^3(7)$

$= \boxed{6x^5 - 24x^4 + 21x^3}$

Ex: $-5x^2y^3(-xy^4 + 8xy - 3x^3y^5 - 2y^2)$

$= -5x^2y^3(-1xy^4) - 5x^2y^3(8xy) - 5x^2y^3(-3x^3y^5) - 5x^2y^3(-2y^2)$

$= \boxed{5x^3y^7 - 40x^3y^4 + 15x^5y^8 + 10x^2y^5}$

Multiplying 2 polynomials with 2 or more terms. $\boxed{5.6.2}$

Recall: distributive property:

$$c(a+b) = ca + cb = ac + bc$$

$$(a+b)(c) = ac + bc$$

Ex: multiply.

$$(\underbrace{3x}_{a} + \underbrace{5}_{b})(\underbrace{x-8}_{c})$$

$$= \underbrace{3x}_{a}\underbrace{(x-8)}_{c} + \underbrace{5}_{b}\underbrace{(x-8)}_{c}$$

$$= 3x^2 - 24x + 5x - 40$$

$$= \boxed{3x^2 - 19x - 40}$$

Ex:

$$(2x-y)(6x-5)$$

$$= 2x(6x-5) - y(6x-5)$$

$$= \boxed{12x^2 - 10x - 6xy + 5y}$$

(no like terms)

Ex:

$$(-2x+8)(x^2+7x-4)$$

$$= -2x(x^2+7x-4) + 8(x^2+7x-4)$$

$$= -2x^3 - 14x^2 + 8x + 8x^2 + 56x - 32$$

$$= \boxed{-2x^3 - 6x^2 + 64x - 32}$$

$\boxed{5.6.3}$

<u>Ex:</u> $(x^2 - 7x - 1)(2x^2 - 5x - 6)$

$= x^2(2x^2 - 5x - 6) - 7x(2x^2 - 5x - 6) - 1(2x^2 - 5x - 6)$

$$
\begin{aligned}
= \; & 2x^4 - 5x^3 - 6x^2 \\
& \quad - 14x^3 + 35x^2 + 42x \\
& \qquad\qquad - 2x^2 + 5x + 6
\end{aligned}
$$

$= \boxed{2x^4 - 19x^3 + 27x^2 + 47x + 6}$

<u>The "FOIL" method</u> (for multiplying two binomials)

F: First

O: Outer

I: Inner

L: Last

<u>Example:</u> $(4x + 3)(5x - 6)$

First Last Inner Outer

$= 20x^2 - 24x + 15x - 18$

$= \boxed{20x^2 - 9x - 18}$

<u>Ex:</u> $(x - 7)(2x - 5)$

$= 2x^2 - 5x - 14x + 35$

$= \boxed{2x^2 - 19x + 35}$

Difference of 2 squares Pattern

$$(a+b)(a-b) = a^2 - b^2$$

Note: $(a+b)(a-b) = a^2 - \cancel{ba} + \cancel{ba} - b^2 = a^2 - b^2$

Ex: $(x+5)(x-5)$

$= x^2 - \cancel{5x} + \cancel{5x} - 25$

$= \boxed{x^2 - 25}$

Perfect Squares

Example: $(x+6)^2$

$= (x+6)(x+6)$

$= x^2 + 6x + 6x + 36$

$= \boxed{x^2 + 12x + 36}$

5.7: Division of Polynomials

Dividing a polynomial by a monomial

Ex: Divide.

$$\frac{6x^4 - 8x^3 + 12x^2 - 9x}{2x^3}$$

$$= \frac{\cancel{6}^{3}x^4}{\cancel{2}x^3} - \frac{\cancel{8}^{4}x^3}{\cancel{2}x^3} + \frac{\cancel{12}^{6}x^2}{\cancel{2}x^3} - \frac{9x}{2x^3}$$

break into separate fractions

Note:

$$\frac{2}{7} + \frac{3}{7}$$
$$= \frac{2+3}{7}$$
$$= \frac{5}{7}$$

$$= \frac{3x}{1} - \frac{4}{1} + \frac{6}{x} - \frac{9}{2x^2}$$

$$= \boxed{3x - 4 + \frac{6}{x} - \frac{9}{2x^2}}$$

Dividing a polynomial by a polynomial
(when divisor has 2 or more terms)

Ex: Divide $\dfrac{3x^2+7x+9}{x+2}$

We will need to do polynomial long division

$$x+2 \overline{\smash{\big)}\,3x^2+7x+9}$$

$\boxed{5.7.2}$

Review:

Numerical example: Divide $\frac{379}{12}$.

$$\begin{array}{r} 31 \\ 12\overline{)379} \\ -\underline{36} \\ 19 \\ -\underline{12} \\ 7 \end{array}$$

So $\frac{379}{12} = 31 + \frac{7}{12}$

$(31)(12) + 7 = 379$

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor}\overline{)\text{Dividend}} \\ \underline{\phantom{\text{Dividend}}} \\ \text{Remainder} \end{array}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$