5.7: Division with Polynomials (cont'd)

$\boxed{5.7.3}$

Note Title 10/16/2017

Example: Divide.

$$\frac{3x^2+7x+9}{x+2}$$

Because the divisor has 2 or more terms, we need to do polynomial long division.

$$\frac{3x^2}{x} \qquad \frac{x}{x}$$

$$\begin{array}{r|llll}
 & 3x & +1 & & \\ \hline
x+2 & 3x^2 & +7x & +9 & \\
 & -(3x^2 & +6x) & & \leftarrow 3x(x+2) = 3x^2+6x \\ \hline
 & \phi & 1x & +9 & \\
 & & -(x & +2) & \leftarrow 1(x+2) = x+2 \\ \hline
 & & \phi & 7 &
\end{array}$$

Write answer:

$$\frac{3x^2+7x+9}{x+2} = \boxed{3x+1+\frac{7}{x+2}}$$

Recall:

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$

Check our answer:

$$(3x+1)(x+2)+7$$

$$= 3x^2+6x+1x+2+7$$

$$= 3x^2+7x+9 \ \checkmark \text{ok}$$

(same as original dividend (numerator))

Example: Divide. $\dfrac{x^3-6x^2+x+14}{x-5}$

$$\frac{x^3}{x} \qquad \frac{-x^2}{x} \qquad \frac{-4x}{x}$$

$$\begin{array}{r|rrrr}
 & x^2 & -x & -4 & \\
\hline
x-5 & x^3 & -6x^2 & +x & +14 \\
 & -(x^3 & -5x^2) & & \\
\hline
 & & -x^2 & +x & \\
 & & -(-x^2 & +5x) & \\
\hline
 & & & -4x & +14 \\
 & & & -(-4x & +20) \\
\hline
 & & & & -6
\end{array}$$

$x^2(x-5) = x^3 - 5x^2$

$-x(x-5) = -x^2 + 5x$

$-4(x-5) = -4x+20$

Write answer:

$$\frac{x^3-6x^2+x+14}{x-5} = \boxed{x^2 - x - 4 + \frac{-6}{x-5}}$$

or $\boxed{x^2 - x - 4 - \dfrac{6}{x-5}}$

Check it:

$$(x-5)(x^2-x-4)-6$$
$$= x(x^2-x-4) - 5(x^2-x-4) - 6$$
$$= x^3 - x^2 - 4x$$
$$\quad - 5x^2 + 5x + 20 - 6$$
$$= x^3 - 6x^2 + x + 14 \checkmark$$

(matches original numerator)

Example: Divide.

$$\frac{8x^3 - 10x^2 + x - 2}{2x - 1}$$

$$\frac{8x^3}{2x} \qquad \frac{-6x^2}{2x} \qquad \frac{-2x}{2x}$$

$$
\begin{array}{r}
4x^2 - 3x - 1 \\
2x - 1 \,\big)\, 8x^3 - 10x^2 + x - 2 \\
-(8x^3 - 4x^2) \\
\hline
-6x^2 + x \\
-(-6x^2 + 3x) \\
\hline
-2x - 2 \\
-(-2x + 1) \\
\hline
-3
\end{array}
$$

$4x^2(2x-1) = 8x^3 - 4x^2$

$-3x(2x-1) = -6x^2 + 3x$

$-1(2x-1) = -2x + 1$

$$\frac{8x^3 - 10x^2 + x - 2}{2x - 1} = \boxed{4x^2 - 3x - 1 + \frac{-3}{2x-1}}$$

$$= \boxed{4x^2 - 3x - 1 - \frac{3}{2x-1}}$$

Check it!

Important: Arrange the dividend and divisor in order of descending powers (biggest power 1st) $\boxed{5.7.6}$

☆ Insert "placeholders" for missing powers of x (ex: $0x^3$, $0x^2$, $0x$ etc)

Example: Divide. $\dfrac{3x^4 - 50x^2 - x}{x-4}$

$\frac{12x^3}{x}$ $\frac{-2x^2}{x}$

$$
\begin{array}{r}
3x^3 + 12x^2 - 2x - 9 \\
x-4 \,\big)\, 3x^4 + 0x^3 - 50x^2 - x + 0 \\
-(3x^4 - 12x^3) \\
\hline
12x^3 - 50x^2 \\
-(12x^3 - 48x^2) \\
\hline
-2x^2 - x \\
-(-2x^2 + 8x) \\
\hline
-9x + 0 \\
-(-9x + 36) \\
\hline
-36
\end{array}
$$

$$\frac{3x^4 - 50x^2 - x}{x-4} = \boxed{3x^3 + 12x^2 - 2x - 9 + \frac{-36}{x-4}}$$

Check it: $(x-4)(3x^3 + 12x^2 - 2x - 9) - 36$

$= 3x^4 + 12x^3 - 2x^2 - 9x$
$\quad - 12x^3 - 48x^2 + 8x + 36 - 36$

$= 3x^4 - 50x^2 - x$ ✓ok

$\boxed{5.7.7}$

Ex: $\dfrac{10x^3+21x^2-5}{2x^2+5x+2}$

$$
\begin{array}{r}
5x-2 \\
2x^2+5x+2\,\overline{\big)\,10x^3+21x^2+0x-5} \\
-(10x^3+25x^2+10x) \\
\hline
-4x^2-10x-5 \\
-(-4x^2-10x-4) \\
\hline
-1
\end{array}
$$

Write answer:

$$\frac{10x^3+21x^2-5}{2x^2+5x+2} = \boxed{5x-2+\frac{-1}{2x^2+5x+2}}$$

$$= \boxed{5x-2-\frac{1}{2x^2+5x+2}}$$

Check it: $(5x-2)(2x^2+5x+2)-1$

$$
\begin{aligned}
&= 10x^3 + 25x^2 + 10x \\
& - 4x^2 - 10x - 4 - 1 \\
&= 10x^3 + 21x^2 - 5 \ \checkmark \text{ ok!}
\end{aligned}
$$