6.2: Factoring Trinomials with Leading Coefficient 1

Note Title 10/25/2017

Example: Factor.

$x^2 + 8x + 15$

GCF: 1

$\boxed{(x+5)(x+3)}$

F: First
O: Outer
I: Inner
L: Last

Check: $(x+5)(x+3)$

$= x^2 + 3x + 5x + 15$

$= x^2 + 8x + 15$ ✓

Also correct: $\boxed{(x+3)(x+5)}$

Example: $x^2 + 10x - 24$

(−) signs are opposite

$\boxed{(x-2)(x+12)}$

If signs are opposite, I want a difference of 10

24
1 · 24
2 · 12
3 · 8
4 · 6

Check: $x^2 + 12x - 2x - 24$

$= x^2 + 10x - 24$ ✓

Also correct: $\boxed{(x+12)(x-2)}$

Ex: $x^2 - 10x + 24$

(+) Signs are the same

$\boxed{(x-6)(x-4)}$

If signs are the same, I want a sum of 10

24
1 · 24
2 · 12
3 · 8
4 · 6

Check:
$x^2 - 4x - 6x + 24$

$= x^2 - 10x + 24$ ✓

6.2.2

(−) signs are opposite

want a difference of 10

Ex: $x^2 - 10x - 24$

$(x + 2)(x - 12)$

Check: $x^2 - 12x + 2x - 24$

$= x^2 - 10x - 24$ ✓

24
1·24
2·12
3·8
4·6

(+) same signs

want a sum of 10

Ex: $x^2 + 10x + 24$

$\boxed{(x + 4)(x + 6)}$

Check: $x^2 + 6x + 4x + 24$

$= x^2 + 10x + 24$ ✓

24
1·24
2·12
3·8
4·6

(−) signs are opposite

want a difference of 16

Ex: $x^2 + 16x - 36$

$\boxed{(x - 2)(x + 18)}$

Check: $x^2 + 18x - 2x - 36$

$= x^2 + 16x - 36$ ✓

36
1·36
2·18
3·12
4·9
6·6

6.2.3

Try these:

a) x^2+6x+9

b) $x^2-3x-54$

c) $x^2+7x-12$

d) $y^2-28y+27$

a) x^2+6x+9 (+) same signs want a sum of 6

9
1·9
3·3

$(x+3)(x+3)$

Check:
$x^2+3x+3x+9$
$=x^2+6x+9$

$\boxed{(x+3)^2}$

b) $x^2-3x-54$ (−) signs are opposite want a difference of 3.

54
1·54
2·27
3·18
6·9

$\boxed{(x+6)(x-9)}$

Check: $x^2-9x+6x-54$
$=x^2-3x-54$ ✓

d) $y^2-28y+27$ (+) same signs want a sum of 28

27
1·27
3·9

$\boxed{(y-1)(y-27)}$

Check: $y^2-27y-1y+27$
$y^2-28y+27$ ✓

$\boxed{6.2.4}$

(c) $x^2 + 7x - 12$ (−) signs are opposite want a difference of 7.

This polynomial is $\boxed{\text{prime.}}$ (cannot be factored)

It's not possible to have a product of 12 and a difference of 7

12
1·12
2·6
3·4

Recall
Prime number: a counting number (larger than 1) that can only be factored (using counting numbers) as a product of 1 and itself.

Ex: 7 is a prime number.
$7(1) = 7$. No other ways to factor it.

Ex: 8 is a composite number (not prime)
$2(4) = 8$

A prime polynomial is a polynomial that can only be factored (using integer coefficients) by factoring out 1 or −1. (prime $\Leftrightarrow$ can't be factored)

Ex: $x^2 - 11xy - 12y^2$ (−) signs are opposite want a difference of 11

$\boxed{(x + 1y)(x - 12y)}$ OR $\boxed{(x+y)(x-12y)}$

12
1·12
2·6
3·4

Check: $x^2 - 12xy + 1xy - 12y^2$
$x^2 - 11xy - 12y^2$

Ex: $2x^2+18x+28$ GCF: 2

$=2(x^2+9x+14)$

(+) same signs

want sum of 9

14
1·14
2·7

$$=\boxed{2(x+2)(x+7)}$$

Check: $(x+2)(x+7)$
$= x^2+7x+2x+14$
$= x^2+9x+14$ ✓

matches

$2(x^2+9x+14)$
$=2x^2+18x+28$ ✓

Important: If your leading coefficient is negative, first factor out −1 (or negative version of your GCF). Then factor what's left.

Ex: $-x^2+14x-40$

$=-1(x^2-14x+40)$

(+) same signs

want a sum of 14

40
1·40
2·20
4·10
5·8

$$=\boxed{-1(x-4)(x-10)}$$

Check: $(x-4)(x-10)$
$=x^2-10x-4x+40$
$=x^2-14x+40$ ✓ (compare to 2nd line)

$-1(x^2-14x+40)$
$=-x^2+14x-40$ ✓

Usually write answer as

$$\boxed{-(x-4)(x-10)}$$

or $\boxed{-(x-10)(x-4)}$