6.5: Difference of Two Squares and Perfect Square Trinomials

Note Title 11/1/2017

Note: 6.4: Factoring Trinomials – the AC method. We are not going to cover the AC method in class. Just work the 6.4 problems the same way as you worked the 6.3 problems. (You are welcome to learn the AC method from the book/videos/tutors, and use it on the test if you like).

Recall: $(a+b)(a-b)$

$= a^2 - ab + ab - b^2 = a^2 - b^2$

Difference of Two Squares Factorization

$$a^2 - b^2 = (a+b)(a-b)$$

Example: Factor.

$x^2 - 25$

$= x^2 - 5^2$ (Use $a^2 - b^2 = (a+b)(a-b)$ with $a = x$, $b = 5$)

$= (x+5)(x-5)$

Check: $x^2 - 5x + 5x - 25$

$= x^2 - 25$

or $(x-5)(x+5)$

Ex: Factor.

$y^2 - 64$

$= y^2 - 8^2$

$= (y+8)(y-8)$

Check:

$y^2 - 8y + 8y - 64$

$= y^2 - 64$

Ex: Factor.

$$4x^2 - 9$$

$$= (2x)^2 - (3)^2$$

$$= \boxed{(2x+3)(2x-3)}$$

Check it!

Perfect Squares:

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$
$13^2 = 169$

Ex: $100y^2 - 81z^2$

$$(10y)^2 - (9z)^2$$

$$\boxed{(10y - 9z)(10y + 9z)}$$

Check:

$$100y^2 + 90yz - 90yz - 81z^2$$

$$= 100y^2 - 81z^2 \quad \checkmark$$

Ex: $x^2 + 49$

Try: $(x+7)(x-7)$

Check: $x^2 - 7x + 7x - 49$

$= x^2 - 49$ No!

This polynomial is **prime.** (does not factor)

Try: $(x-7)(x-7)$

check: $x^2 - 7x - 7x + 49$

$= x^2 - 14x + 49$ No!

Try: $(x+7)(x+7)$

check: $x^2 + 7x + 7x + 49$

$= x^2 + 14x + 49$

Important Fact:

$x^2 +$ positive is always prime. (does not factor)

$a^2 + b^2$ is prime. (does not factor). (assuming GCF has been factored out)

Why is $x^2 + 49$ prime?

write as a trinomial: $x^2 + 0x + 49$

(+) same signs want a sum of $0x$

x^2 : $x \cdot x$; 49 : $1 \cdot 49$, $7 \cdot 7$

It's impossible to get a sum of 0, so it is **prime.**

Note: $x^2 - 49$

$x^2 + 0x - 49$

(−) signs are opposite. Want a difference of 0

$(x + 7)(x - 7)$

x^2 : $x \cdot x$; 49 : $1 \cdot 49$, $7 \cdot 7$; $7x$, $7x$

Ex: Factor. $32x^2 - 50y^2$

$= 2(16x^2 - 25y^2)$

$= 2((4x)^2 - (5y)^2)$ (use $a = 4x$, $b = 5y$)

$= 2(4x + 5y)(4x - 5y)$

Check it!

Ex: $x^5 - x$

$= x(x^4 - 1)$

$= x((x^2)^2 - (1)^2)$

$= x(x^2+1)(x^2-1)$

$= \boxed{x(x^2+1)(x+1)(x-1)}$

Check

$(x+1)(x-1)$

$= x^2 - 1x + 1x - 1$

$= x^2 - 1$ ✓

Ex: $x^4 - 13x^2 + 36$

(+) same signs, sum of $13x^2$ for middle term

x^4 ∧ $x^2 \cdot x^2$ — $9x^2$ — $4x^2$

36 ∧
1 · 36
2 · 18
3 · 12
4 · 9
6 · 6

$(x^2 - 4)(x^2 - 9)$

$\boxed{(x+2)(x-2)(x+3)(x-3)}$

Ex: $-x^2 + 36$

$= -1(x^2 - 36)$

$= -1(x+6)(x-6)$

$= \boxed{-(x+6)(x-6)}$

Perfect Square Trinomials

Note: $(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

Check: $(a+b)(a+b)$

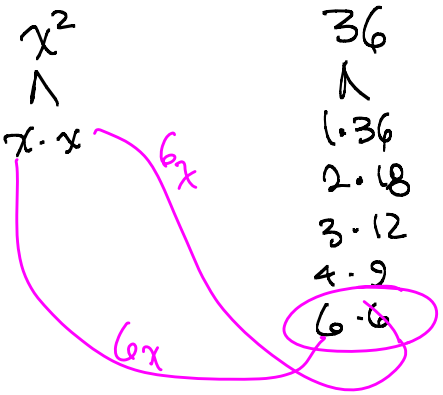
$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$ ✓

Ex: Factor.

$x^2 + 12x + 36$ (+) same signs sum of 12x

$= (x + 6)(x + 6)$



Check: $x^2 + 6x + 6x + 36$

$= x^2 + 12x + 36$ ✓

$\boxed{(x+6)^2}$

Ex: Factor.

$16x^2 - 56xy + 49y^2$

Try: $(4x - 7y)^2$

Check: $(4x - 7y)(4x - 7y)$

$16x^2 - 28xy - 28xy + 49y^2$

$= 16x^2 - 56xy + 49y^2$ ok!

$\boxed{(4x-7y)^2}$

Ex: Factor. $4x^2 - 20x + 9$

Try: $(2x - 3)^2$

Check: $(2x-3)(2x-3)$

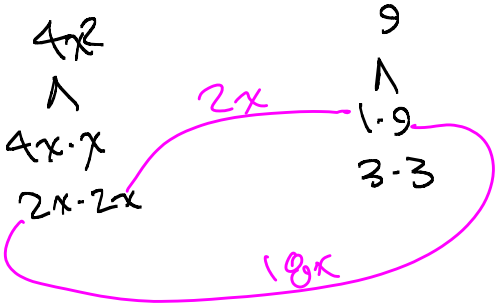
$= 4x^2 - 6x - 6x + 9$

$= 4x^2 - 12x + 9$ No!

Start over.

$4x^2 - 20x + 9$ (+) same signs sum of 20x

$\boxed{(2x - 1)(2x - 9)}$



Check: $4x^2 - 18x - 2x + 9$

$4x^2 - 20x + 9$ ✓

7.1: Simplifying Rational Expressions

7.1.1

Recall:

Rational Number: Can be written as the ratio (fraction) of two integers.

Ex: $\frac{3}{2}$, $-\frac{7}{4}$, $5 = \frac{5}{1}$

Rational Expression: Can be written as the ratio (fraction) of two polynomials.

Simplifying a rational expression means reducing the fraction by dividing out ("canceling") factors that appear in both the numerator and denominator.

Ex: $\frac{8}{20} = \frac{4 \cdot 2}{4 \cdot 5} = \boxed{\frac{2}{5}}$

Ex: Simplify. $\frac{24x^4y^3}{32x^5y} = \boxed{\frac{3y^2}{4x}}$

Ex: Simplify. $\frac{x^2+8x+15}{x^2+5x+6} = \frac{(x+3)(x+5)}{(x+3)(x+2)} = \boxed{\frac{x+5}{x+2}}$

Can I cancel the x's? No! Can only cancel factors, not terms

Restrictions:

$x \neq -3$

$x \neq -2$

Ex: Simplify. $\dfrac{x-3}{9-x^2}$.

$$\frac{x-3}{-x^2+9}$$

$$= \frac{x-3}{-1(x^2-9)} = \frac{x-3}{-1(x+3)(x-3)}$$

$$= \frac{1}{-1(x+3)} = \boxed{-\frac{1}{x+3}}$$

Note: $3-x = -(x-3)$