

7.1: Simplifying Rational Expressions (cont'd): 7.1.3

Note Title

11/6/2017

Ex: Simplify.

$$\frac{x-2}{x^2-6x+8}$$

$$= \frac{\cancel{x-2}}{\cancel{(x-2)}(x-4)} = \frac{1}{x-4}$$

Note: $\frac{8}{24} = \frac{1}{3}$

Scratchwork
 $x^2 - 6x + 8$
 $(x-2)(x-4)$
 (+) same signs
 sum of 6

Check:
 $x^2 - 4x - 2x + 8$
 $x^2 - 6x + 8 \checkmark$

Ex: Simplify.

$$\frac{8x^2+10x+3}{8x^2+2x-3}$$

$$= \frac{(2x+1)(4x+3)}{(2x-1)(4x+3)}$$

$$= \frac{2x+1}{2x-1}$$

Restricted values: $2x-1 \neq 0$
 $2x \neq 1$

$$\frac{2x}{2} \neq \frac{1}{2}$$

$$4x+3 \neq 0$$

$$4x \neq -3$$

$$x \neq -\frac{3}{4}$$

Scratchwork

$$8x^2+10x+3$$

(+) same signs
 want sum of 10x

$$(2x+1)(4x+3)$$

Check:

$$\begin{array}{r} 8x^2 \\ \uparrow \\ x \cdot 8x \\ 2x \cdot 4x \end{array}$$

4x
 6x

$$8x^2+6x+4x+3$$

$$8x^2+10x+3 \checkmark$$

$$8x^2+2x-3$$

(-) opp. signs
 want a difference of 2x

$$(2x-1)(4x+3)$$

Check:

$$8x^2+6x-4x-3$$

$$8x^2+2x-3 \checkmark$$

$$\begin{array}{r} 8x^2 \\ \uparrow \\ x \cdot 8x \\ 2x \cdot 4x \end{array}$$

4x
 6x

Ex:

Simplify.

$$\frac{x^2 - x - 12}{4x^2 + 12x} = \frac{(x+3)(x-4)}{4x(x+3)}$$

$$= \boxed{\frac{x-4}{4x}}$$

Restricted
values:

$$\boxed{\begin{array}{l} x \neq 0 \\ x \neq -3 \end{array}}$$

$$x+3 \neq 0$$

$$x \neq -3$$

Ex:

$$\frac{25-x^2}{x+5} = \frac{-x^2+25}{x+5}$$

$$= \frac{-1(x^2-25)}{x+5} = \frac{-1(x+5)(x-5)}{x+5} = \frac{-1(x-5)}{1}$$

$$\begin{aligned} &= -1(x-5) \\ &= \boxed{-(x-5)} \end{aligned}$$

7.2: Multiplying and Dividing Rational Expressions

7.2.1

Recall: $\frac{2}{3} \cdot \frac{4}{5} = \frac{2(\cancel{4})}{3(\cancel{5})} = \boxed{\frac{8}{15}}$

Ex: $\frac{\cancel{16}^1}{\cancel{15}_5} \cdot \frac{\cancel{21}^7}{\cancel{32}_2} = \boxed{\frac{7}{16}}$

Ex: $\frac{3x^2y}{\cancel{4z}^1} \cdot \frac{\cancel{16}^4 z^5}{xy^3} = \frac{12x^2y z^5}{xy^3 z} = \boxed{\frac{12xz^4}{y^2}}$

Ex: $\frac{x^2+2x}{x^2-4} \cdot \frac{x^2-4x+4}{x^2-x}$ (+)
same signs
sum of 4 Factor all the
numerators and denominators

$= \frac{x(\cancel{x+2})}{(\cancel{x+2})(\cancel{x-2})} \cdot \frac{(\cancel{x-2})(x-2)}{x(x-1)} = \frac{x(\cancel{x+2})(\cancel{x-2})(x-2)}{(\cancel{x+2})(\cancel{x-2})(x)(x-1)}$

$= \frac{\cancel{x}(x-2)}{\cancel{x}(x-1)} = \boxed{\frac{x-2}{x-1}}$

Ex: $(x+2) \cdot \frac{5}{4x^2-16}$

$= \frac{x+2}{1} \cdot \frac{5}{4(\cancel{x+2})(x-2)} = \frac{\cancel{x+2}}{1} \cdot \frac{5}{4(\cancel{x+2})(x-2)}$

$= \boxed{\frac{5}{4(x-2)}} = \boxed{\frac{5}{4x-8}}$

Division of Rational Expressions

7.2.2

Recall: $\frac{3}{4} \div \frac{5}{7}$

$$= \frac{3}{4} \cdot \frac{7}{5}$$

$$= \boxed{\frac{21}{20}}$$

Recall: Dividing by a number is the same as multiplying by its reciprocal.

Ex.: $\frac{x^2+2x}{x+5} \div \frac{4-x^2}{3x-6}$

$$= \frac{x^2+2x}{x+5} \cdot \frac{3x-6}{-x^2+4}$$

$$= \frac{x(x+2)}{x+5} \cdot \frac{3(x-2)}{-1(x^2-4)}$$

$$= \frac{x \cancel{(x+2)}}{x+5} \cdot \frac{3 \cancel{(x-2)}}{-1 \cancel{(x+2)} \cancel{(x-2)}}$$

[change to multiplication]

$$= \frac{3x}{-1(x+5)} = \frac{3x}{-(x+5)}$$
$$= \boxed{-\frac{3x}{x+5}}$$

$$= \frac{3x}{(x+5)(-1)}$$

$$= \boxed{\frac{3x}{-x-5}}$$

7.2.3

Ex: $\frac{x^2 + 7x + 12}{x^2 - 4x} \div (x^2 - 9)$

$$= \frac{x^2 + 7x + 12}{x^2 - 4x} \div \frac{x^2 - 9}{1}$$

$$= \frac{x^2 + 7x + 12}{x^2 - 4x} \cdot \frac{1}{x^2 - 9}$$

$$= \frac{\cancel{(x+3)}(x+4)}{x(x-4)} \cdot \frac{1}{\cancel{(x+3)}(x-3)}$$

$$= \boxed{\frac{x+4}{x(x-4)(x-3)}}$$

$$= \frac{x+4}{x(x^2 - 7x + 12)}$$

$$= \boxed{\frac{x+4}{x^3 - 7x^2 + 12x}}$$

[change to multiplication]

[factor all numerators and denominators]

Ex: $\frac{x^2 - 6x}{x+8} \cdot \frac{x}{6-x}$

$$= \frac{x(x-6)}{x+8} \cdot \frac{x}{6-x}$$

$$= \frac{x\cancel{(x-6)}}{x+8} \cdot \frac{x}{-1\cancel{(x-6)}}$$

$$= \frac{x^2}{(x+8)(-1)}$$

$$= \frac{x^2}{-(x+8)} = \boxed{-\frac{x^2}{x+8}}$$

Note: $x-6 \neq 6-x$
 $6-x = -x+6$
 $= -1(x-6)$
 $6-x$ and $x-6$ are opposite

7.4: Addition and Subtraction of Rational Expressions

(7.4.1)

Recall:

multiplication: $\frac{3}{5} \cdot \frac{4}{5} = \frac{3 \cdot 4}{5 \cdot 5} = \boxed{\frac{12}{25}}$

Addition:
(same denominator) $\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \boxed{\frac{7}{5}}$

Addition:
(different denominators)

$$\frac{3}{5} + \frac{2}{7}$$

LCD: 35

$$\frac{3}{5} \left(\frac{7}{7} \right) + \frac{2}{7} \left(\frac{5}{5} \right)$$

$$\frac{21}{35} + \frac{10}{35} = \boxed{\frac{31}{35}}$$

Ex.:

$$\begin{aligned} & \frac{1}{6x} + \frac{7}{6x} \\ &= \frac{1+7}{6x} = \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{6}x}} = \boxed{\frac{4}{3x}} \end{aligned}$$

Ex.:

$$\begin{aligned} & \frac{3}{x+5} - \frac{10}{x+5} \\ &= \frac{3-10}{x+5} \\ &= \boxed{\frac{-7}{x+5}} = \boxed{-\frac{7}{x+5}} \end{aligned}$$

7.4.2

Ex: $\frac{3y-5}{y+4} - \frac{5y-7}{y+4}$

$$= \frac{3y-5}{y+4} - \frac{(5y-7)}{y+4}$$

$$= \frac{3y-5 - (5y-7)}{y+4} = \frac{3y-5-5y+7}{y+4}$$

$$= \boxed{\frac{-2y+2}{y+4}} = \boxed{\frac{-2(y-1)}{y+4}}$$

Ex: $\frac{2x+1}{x^2-1} + \frac{x+2}{x^2-1}$

$$= \frac{2x+1+x+2}{x^2-1} = \frac{3x+3}{x^2-1}$$

$$= \frac{3(x+1)}{(x+1)(x-1)} = \boxed{\frac{3}{x-1}}$$

Ex: $\frac{7x}{30} - \frac{11x}{12}$ LCD: 60

$$= \frac{7x}{30} \left(\frac{2}{2}\right) - \frac{11x}{12} \left(\frac{5}{5}\right)$$

$$= \frac{14x}{60} - \frac{55x}{60} = \boxed{\frac{-41x}{60}}$$

7.4.3

Ex.: $\frac{5y}{6x^2} - \frac{y}{4x}$

LCD: $12x^2$

$= \frac{5y}{6x^2} \left(\frac{2}{2} \right) - \frac{y}{4x} \left(\frac{3x}{3x} \right)$

$= \frac{10y}{12x^2} - \frac{3xy}{12x^2} = \frac{10y - 3xy}{12x^2} = \frac{y(10 - 3x)}{12x^2}$

Ex.: $\frac{2}{xy^4} + \frac{3}{x^2y^2}$

LCD: x^2y^4

$= \frac{2}{xy^4} \left(\frac{x}{x} \right) + \frac{3}{x^2y^2} \left(\frac{y^2}{y^2} \right)$

$= \frac{2x}{x^2y^4} + \frac{3y^2}{x^2y^4} = \frac{2x + 3y^2}{x^2y^4}$

Ex.: $\frac{3}{x-3} - \frac{6}{x+5}$

LCD: $(x-3)(x+5)$

$= \frac{3}{x-3} \left(\frac{x+5}{x+5} \right) + \frac{-6}{x+5} \left(\frac{x-3}{x-3} \right)$

$= \frac{3x+15-6x+18}{(x-3)(x+5)} = \frac{-3x+33}{(x-3)(x+5)} = \frac{-3(x-11)}{(x-3)(x+5)}$

Ex.: $\frac{2x}{x-4} + \frac{5x}{4-x}$

$$= \frac{2x}{x-4} + \frac{5x}{-x+4}$$

$$= \frac{2x}{x-4} + \frac{5x}{-1(x-4)}$$

$$= \frac{2x}{x-4} - \frac{5x}{x-4}$$

$$= \frac{2x-5x}{x-4} = \boxed{\frac{-3x}{x-4}}$$

7.4.4

Note:

$$4-x = -(x-4)$$