7.7: Applications of Rational Equations and Proportions

7.7.1

Note Title 11/13/2017

Example: A cookie recipe requires 6 cups of flour for 4 dozen cookies. How many cups of flour are needed to make 10 dozen cookies?

$$\frac{6 \text{ cups}}{4 \text{ doz}} = \frac{?}{10 \text{ doz}}$$

6 cups flour to 4 dozen cookies is the same as ? cups of flour to 10 dozen cookies

$$\frac{6 \text{ cups}}{4 \text{ doz}} = \frac{x}{10 \text{ doz}}$$

without units:

$$\frac{6}{4} = \frac{x}{10}$$

$$\left(\frac{10 \cancel{\text{doz}}}{1}\right)\frac{6 \text{ cups}}{4 \cancel{\text{doz}}} = \frac{x}{\cancel{10 \text{ doz}}}\left(\frac{\cancel{10 \text{ doz}}}{1}\right)$$

$$\left(\frac{10}{1}\right)\frac{6}{4} = \frac{x}{10}\left(\frac{10}{1}\right)$$

$$\frac{60}{4} = x$$

$$\underline{15} = x$$

$$\frac{60 \text{ cups}}{4} = x$$

$$15 \text{ cups} = x$$

$\boxed{\text{So 15 cups of flour are needed.}}$

Ex.: A car gets 30 miles to the gallon. How much gas is needed to drive 150 miles?

$$\frac{30 \text{ mi}}{1 \text{ gal}} = \frac{150 \text{ mi}}{x}$$

OR

$$\frac{1 \text{ gal}}{30 \text{ mi}} = \frac{x}{150 \text{ mi}}$$

cross-multiplying:

$$30x = 150$$

$$\frac{30x}{30} = \frac{150}{30}$$

$$x = 5$$

$$\left(\frac{150 \cancel{\text{mi}}}{1}\right)\frac{1 \text{ gal}}{30 \cancel{\text{mi}}} = \frac{x}{\cancel{150 \text{ mi}}}\left(\frac{\cancel{150 \text{ mi}}}{1}\right)$$

$$\frac{150 \text{ gal}}{30} = x$$

$$5 \text{ gal} = x$$

$\boxed{\text{5 gallons gas are needed}}$

$\boxed{7.7.2}$

Ex: Solve. $\frac{15}{2} = \frac{7}{4x}$ LCD: $4x$

$$\left(\frac{2x}{2x}\right)\frac{15}{2} = \frac{7}{4x}$$

$$\frac{30x}{4x} = \frac{7}{4x}$$

$$\left(\frac{\cancel{4x}}{1}\right)\cdot\frac{30x}{\cancel{4x}} = \frac{7}{\cancel{4x}}\cdot\left(\frac{\cancel{4x}}{1}\right)$$

$$30x = 7$$

$$\frac{30x}{30} = \frac{7}{30}$$

$$x = \frac{7}{30}$$

Sol'n Set: $\left\{\frac{7}{30}\right\}$

Ex: $\frac{6}{5} = \frac{12}{y}$ LCD: $5y$

$$\left(\frac{y}{y}\right)\frac{6}{5} = \frac{12}{y}\left(\frac{5}{5}\right)$$

$$\frac{6y}{5y} = \frac{60}{5y}$$

$$\frac{\cancel{5y}}{1}\cdot\frac{6y}{\cancel{5y}} = \frac{60}{\cancel{5y}}\cdot\frac{\cancel{5y}}{1}$$

$$6y = 60$$

$$\frac{6y}{6} = \frac{60}{6}$$

$$y = 10$$

Sol'n Set: $\boxed{\{10\}}$

Cross-multiplication

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

$$\frac{a}{b} = \frac{c}{d}$$

Ex: Solve.

$$\frac{y+2}{8} = \frac{2y}{5}$$

$$5(y+2) = 8(2y)$$

$$5y + 10 = 16y$$

$$-5y \qquad -5y$$

$$10 = 11y$$

$$\frac{10}{11} = \frac{11y}{11}$$

$$\frac{10}{11} = y$$

Sol'n Set

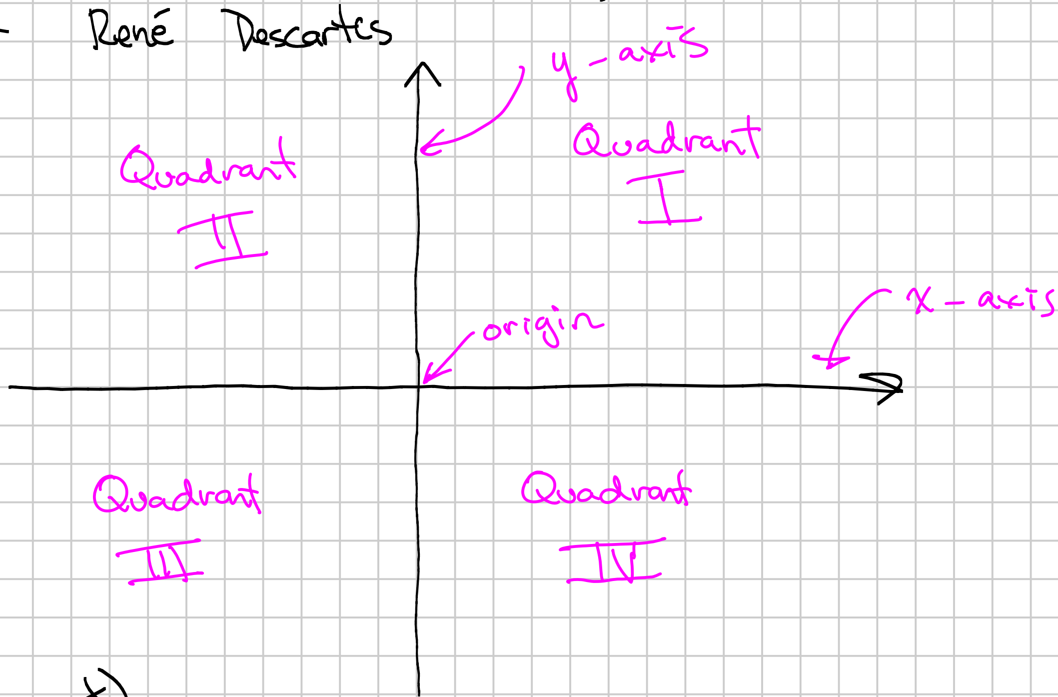
$$\boxed{\left\{ \frac{10}{11} \right\}}$$

3.1: The Rectangular Coordinate System

Note Title 2/5/2015

(also called the Cartesian coordinate system, the Cartesian plane, the xy-plane)

Named after René Descartes



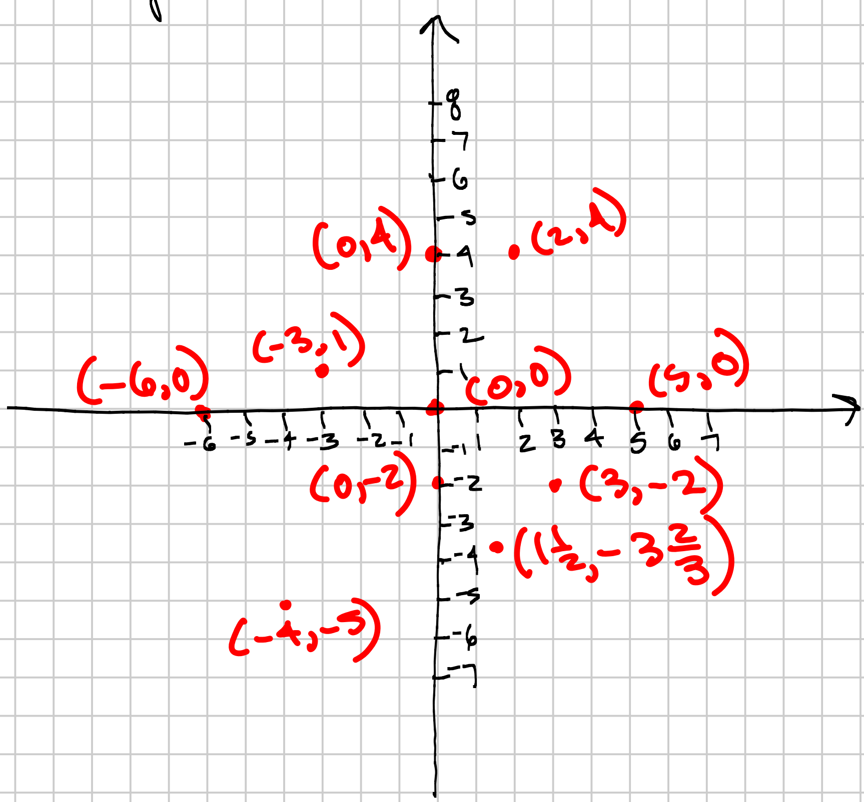
Each location (point) on the xy-plane is described by an <u>ordered pair</u>, written in the form (x,y).

1st coordinate = x-coordinate
2nd coordinate = y-coordinate

<u>Ex</u>: Plot these points

$(2,4)$ $\qquad$ $(0,0)$
$(-3,1)$ $\qquad$ $(0,4)$
$(-4,-5)$ $\qquad$ $(5,0)$
$(3,-2)$ $\qquad$ $(-6,0)$
$\left(\frac{3}{2}, -\frac{11}{3}\right)$ $\qquad$ $(0,-2)$

Rewrite: $\left(1\frac{1}{2}, -3\frac{2}{3}\right)$



3.2: Linear Equations in Two Variables

A linear equation in 2 variables can be written as
$Ax+By=C$, where A, B, C are real numbers.

A solution to a linear equation in two variables is an ordered pair that makes the equation true.
(when we substitute the 1st coordinate for x and the 2nd coordinate for y).

Ex: Is $(4,6)$ a solution to $3x-2y=1$?

$x=4,\ y=6$

$3(4)-2(6)=1$
$12-12=1$
$0=1$ False!

$\boxed{\text{No, } (4,6) \text{ is not a solution.}}$

Is $(5,7)$ a solution to $3x-2y=1$?

$x=5,\ y=7$

$3(5)-2(7)=1$
$15-14=1$
$1=1$ True! ✓

$\boxed{\text{Yes, } (5,7) \text{ is a solution.}}$

Is $(10,14)$ a solution?

$x=10,\ y=14$

$3x-2y=1$
$3(10)-2(14)=1$
$30-28=1$
$2=1$ False

No, $(10,14)$ is not a solution.

Is $(3,4)$ a solution?

$x=3,\ y=4$

$3(3)-2(4)=1$
$9-8=1$
$1=1$ ✓ True!

$\boxed{\text{Yes, } (3,4) \text{ is a solution.}}$

Is $(1,1)$ a solution?

$3x - 2y = 1$
$3(1) - 2(1) = 1$
$3 - 2 = 1$
$1 = 1$ ✓

Yes, $(1,1)$ is a solution.

Is $(7,10)$ a solution?

$3(7) - 2(10) = 1$
$21 - 20 = 1$
$1 = 1$ True!

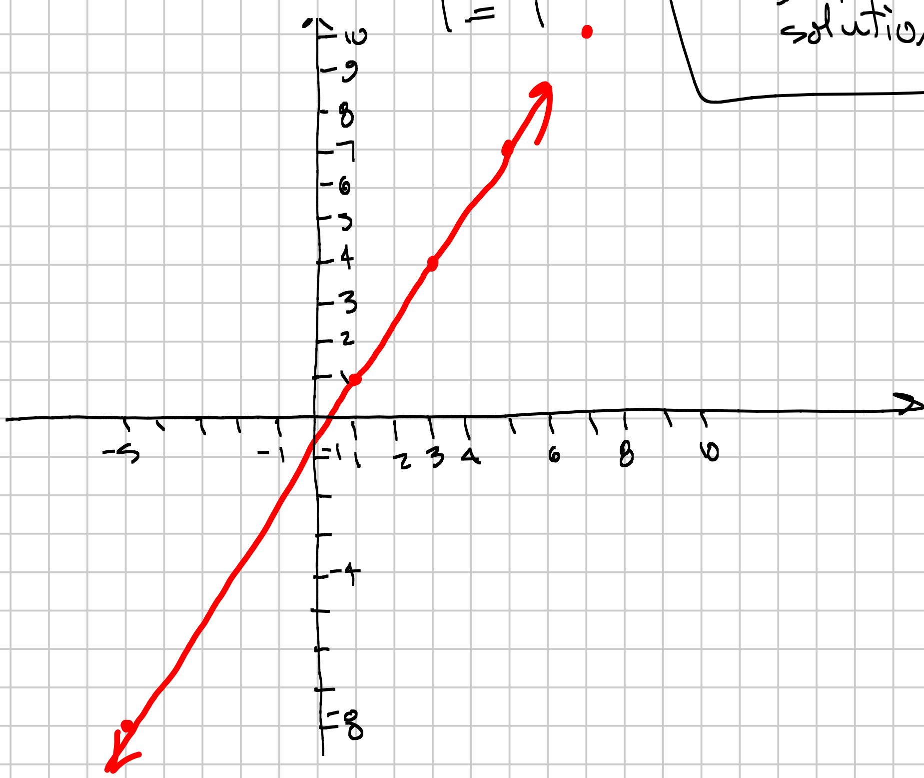
Yes, $(7,10)$ is a solution.

Is $(-5,-8)$ a solution?

$3(-5) - 2(-8) = 1$
$-15 + 16 = 1$
$1 = 1$

Yes, $(-5,-8)$ is a solution.

Points:
$(1,1)$
$(3,4)$
$(5,7)$
$(-5,-8)$
$(7,10)$



The graph of an equation is the set of all points that make the eqn true.

Ex: Graph the line $-3x+2y=-4$. Use the given coordinates to create solutions.

x	y
2	
4	
	5
	-2

x	y
2	1
4	4
$4\frac{2}{3}$	5
0	-2

$x=2$)

$-3x+2y=-4$

$-3(2)+2y=-4$

solve for y:

$-6+2y=-4$

$+6 \qquad +6$

$2y=2$

$\frac{2y}{2}=\frac{2}{2}$

$y=1$

$(2,1)$

$x=4$)

$-3x+2y=-4$

$-3(4)+2y=-4$

$-12+2y=-4$

$+12 \qquad +12$

$2y=8$

$y=\frac{8}{2}=4$

$(4,4)$

$y=5$)

$-3x+2y=-4$

$-3x+2(5)=-4$

$-3x+10=-4$

$-10 \qquad -10$

$-3x=-14$

$\frac{-3x}{-3}=\frac{-14}{-3}$

$x=\frac{14}{3}=4\frac{2}{3}$

$(4\frac{2}{3},5)$

$y=-2$)

$-3x+2y=-4$

$-3x+2(-2)=-4$

$-3x-4=-4$

$+4 \qquad +4$

$-3x=0$

$\frac{-3x}{-3}=\frac{0}{-3}$

$x=0$

$(0,-2)$

$(4\frac{2}{3},5)$

$(0,-2)$

$(2,1)$

$(4,4)$