4.2: Solving linear systems with the Substitution Method

Note Title 11/29/2017

Solving by substitution (step-by-step)

1) Solve one equation (your choice) for one variable (your choice).

2) Substitute this expression into the other equation.

3) Solve for the remaining variable.

4) Plug this value into either equation and solve.

5) Check your answer.

Example: Solve the system by substitution.

$$\begin{cases} x - y = 9 \\ 2x - 11y = -18 \end{cases}$$

1) Solve $x - y = 9$ for x:

$$\begin{array}{rcl} x - y &=& 9 \\ +y & & +y \end{array}$$

$$x = y + 9$$

2) Substitute $x = y + 9$ into $2x - 11y = -18$.

$$2(y + 9) - 11y = -18$$

3) solve for y:

$$2y + 18 - 11y = -18$$

$$\begin{array}{rcl} -9y + 18 &=& -18 \\ -18 & & -18 \end{array}$$

$$-9y = -36$$

$$\frac{-9y}{-9} = \frac{-36}{-9}$$

$$y = 4$$

4) Plug $y = 4$ into $x - y = 9$ and solve:

$$y = 4 \Rightarrow \begin{array}{rcl} x - 4 &=& 9 \\ +4 & & +4 \\ x &=& 13 \end{array}$$

Solution: $\boxed{(13, 4)}$

or solution set: $\boxed{\{(13, 4)\}}$

5) Check your answer.

$$x - y = 9$$

$$x = 13,\ y = 4 \Rightarrow 13 - 4 = 9$$

$$9 = 9 \ \checkmark \text{True!}$$

$$2x - 11y = -18$$

$$x = 13,\ y = 4 \Rightarrow 2(13) - 11(4) = -18$$

$$26 - 44 = -18$$

$$-18 = -18 \ \checkmark \text{True!}$$

Ex: Solve by substitution.

4.2.3

$$-4x + 2y = -4$$
$$2x + y = 8$$

1) Solve $2x+y=8$ for y:

$$\begin{aligned} 2x + y &= 8 \\ -2x \quad & \quad -2x \end{aligned}$$

$$y = 8 - 2x$$

2) Substitute $y = 8-2x$ into $-4x+2y=-4$.

$$-4x + 2(8-2x) = -4$$

3) Solve for x:

$$-4x + 16 - 4x = -4$$
$$\begin{aligned} -8x + 16 &= -4 \\ -16 \quad & \quad -16 \end{aligned}$$
$$-8x = -20$$
$$\frac{-8x}{-8} = \frac{-20}{-8}$$
$$x = \frac{\cancel{20}^{\,5}}{\cancel{8}_{\,2}} = \frac{5}{2} = 2\tfrac{1}{2}$$

4) Plug $x = \frac{5}{2}$ into $2x+y=8$ and solve:

$$\frac{2}{1}\left(\frac{5}{2}\right) + y = 8$$
$$\frac{10}{2} + y = 8$$
$$\begin{aligned} 5 + y &= 8 \\ -5 \quad & \quad -5 \end{aligned}$$
$$y = 3$$

Solution:

$$\boxed{\left(\frac{5}{2}, 3\right)}$$

5) Check:

$2x + y = 8$

$x = \frac{5}{2}, y = 3 \Rightarrow \frac{2}{1}\left(\frac{5}{2}\right) + 3 = 8$

$\frac{10}{2} + 3 = 8$

$5 + 3 = 8$

$8 = 8$ ✓ true!

$-4x + 2y = -4$

$x = \frac{5}{2}, y = 3 \Rightarrow -\frac{4}{1}\left(\frac{5}{2}\right) + 2(3) = -4$

$-\frac{20}{2} + 6 = -4$

$-10 + 6 = -4$

$-4 = -4$

✓ true!

4.3: Solving systems by the elimination/addition method

Steps for solving by elimination/addition

1) Choose a variable to eliminate, then multiply one or both equations by a strategic number, so that when you add the equations, the variable will disappear.
   (look for the Least Common Multiple of the coefficients)

2) Add the equations. One variable should disappear.

3) Solve for the remaining variable.

4) Either (a) Repeat steps 1-3 for the other variable,
   or (b) Plug your value into either of the original equations and solve.

5) Check your solution.

Ex: Solve the system using the elimination method. (addition method). $\boxed{4.3.2}$

$$\begin{cases} 11x - 5y = 2 \\ 3x - y = 1 \end{cases}$$

$$\begin{array}{ll} 11x - 5y = 2 \longrightarrow & 11x - 5y = 2 \\ 3x - y = 1 \xrightarrow{(-5)} & -15x + 5y = -5 \\ \hline \text{Add:} & -4x + 0 = -3 \end{array}$$

$$-4x = -3$$

$$\frac{-4x}{-4} = \frac{-3}{-4}$$

$$x = \frac{3}{4}$$

Could plug $x = \frac{3}{4}$ into either eqn, or do elimination again to get 2nd variable:

$$\begin{array}{ll} 11x - 5y = 2 \xrightarrow{(-3)} & -33x + 15y = -6 \\ 3x - y = 1 \xrightarrow{(11)} & 33x - 11y = 11 \\ \hline \text{Add:} & 0 + 4y = 5 \end{array}$$

$$4y = 5$$

$$\frac{4y}{4} = \frac{5}{4}$$

$$y = \frac{5}{4}$$

Solution:

$$\boxed{\left(\frac{3}{4}, \frac{5}{4}\right)}$$

Check your solution:

$$11x - 5y = 2$$

$$x = \frac{3}{4}, y = \frac{5}{4} \Rightarrow \frac{11}{1}\left(\frac{3}{4}\right) - \frac{5}{1}\left(\frac{5}{4}\right) = 2$$

$$\frac{33}{4} - \frac{25}{4} = 2$$

$$\frac{8}{4} = 2$$

$$2 = 2 \checkmark \text{True}$$

$$3x - y = 1$$

$$x = \frac{3}{4}, y = \frac{5}{4} \Rightarrow \frac{3}{1}\left(\frac{3}{4}\right) - \frac{5}{4} = 1$$

$$\frac{9}{4} - \frac{5}{4} = 1$$

$$\frac{4}{4} = 1$$

$$1 = 1 \checkmark \text{True!}$$

Ex: Solve the system by elimination. $\boxed{4.3.3}$

$$\begin{cases} -5x + 2y = -6 \\ 7y + 10x = 34 \end{cases}$$

$$\begin{array}{lll} -5x+2y=-6 & \longrightarrow -5x+2y=-6 & \xrightarrow{(2)} -10x+4y=-12 \\ 7y+10x=34 & \longrightarrow 10x+7y=34 & \longrightarrow 10x+7y=34 \end{array}$$

Add: $0 + 11y = 22$

$$\frac{11y}{11} = \frac{22}{11}$$

$$y = 2$$

2 options:
can put $y=2$ into $-5x+2y=-6$

$$-5x+2(2)=-6$$
$$-5x+4=-6$$
$$-4 \quad -4$$
$$-5x=-10$$
$$\frac{-5x}{-5} = \frac{-10}{-5}$$
$$x=2$$

Sol'n: $\boxed{(2,2)}$

or Do elimination a 2nd time to find x:

$$\begin{array}{ll} -5x+2y=-6 & \xrightarrow{(-7)} 35x-14y=42 \\ 10x+7y=34 & \xrightarrow{(2)} 20x+14y=68 \end{array}$$

Add: $55x + 0 = 110$

$$55x = 110$$
$$\frac{55x}{55} = \frac{110}{55}$$
$$x = 2$$

Sol'n: $\boxed{(2,2)}$

Check it!

4.3.4

Ex: Solve the system.

$$\begin{cases} -x+6y=2 \\ 3x-18y=5 \end{cases}$$

$$\begin{array}{rcl} -x+6y=2 & \xrightarrow{(3)} & -3x+18y=6 \\ 3x-18y=5 & \longrightarrow & 3x-18y=5 \\ \hline & \text{Add:} & 0+0=11 \end{array}$$

$0=11$ False!

Substitution

Solve $-x+6y=2$ for x:

$-6y \quad -6y$

$$-x=2-6y$$

$$\frac{-x}{-1}=\frac{2}{-1}-\frac{6y}{-1}$$

$$x=-2+6y$$

Substitute $x=-2+6y$ into $3x-18y=5$

$$3(-2+6y)-18y=5$$

$$-6+\cancel{18y}-\cancel{18y}=5$$

$-6=5$ False

$+6 \quad +6$

$0=11$ False

False statement, so $\boxed{\text{No solution}}$. This is an $\boxed{\text{inconsistent system.}}$ (Lines are parallel)

Ex: Solve.

$$2x - 10y = 6$$
$$-5x + 25y = -15$$

$$2x - 10y = 6 \xrightarrow{(5)} 10x - 50y = 30$$
$$-5x + 25y = -15 \xrightarrow{(2)} -10x + 50y = -30$$

Add: $0 + 0 = 0$

$0 = 0$ True!

True, so this is a $\boxed{\text{dependent system}}$ with $\boxed{\text{infinitely many solutions}}$ (lines are the same)

Summary:

When solving a system by elimination or substitution and both variables disappear:

1) If you get a false statement (such as $0 = 11$ or $5 = -6$), the system is inconsistent & has no solution (lines are parallel)

2) If you get a true statement (such as $0 = 0$ or $3 = 3$), the system is dependent & has infinitely many solutions (lines are the same)

If you get an ordered pair solution, the system is independent.