5.2: The Mean and Standard Deviation of a Discrete Random Variable

Mean of a discrete random variable:

> The Mean (Expected value) of a Discrete Random Variable:
>
> Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of X is
>
> $$\mu = x_1 p_1 + x_2 p_2 + ... + x_n p_n.$$
>
> Suppose an experiment is repeated many times, and the values of X are recorded and then averaged. As the number of repetitions increases, the average value of X will become closer and closer to μ. For that reason, the mean is called the *expected value* of X.

Example 1: A probability distribution is given by the table below. Find the mean (the expected value of X).

x	3	4	5	6	7	8	9
$P(X = x)$	0.15	0.20	0.30	0.12	0.08	0.10	0.05

Mean: $\mu = E(X) = 3(0.15) + 4(0.20) + 5(0.30) + 6(0.12)$
$+ 7(0.08) + 8(0.10) + 9(0.05) = \boxed{5.28 = \mu}$

Example 2: Suppose that an organization sells 1000 raffle tickets for $1 each. One ticket ~~is for~~ holder will win a gift basket worth $200, and three tickets ~~are for~~ holders will win $50 gift certificates to a restaurant. Find the expected net winnings for a person who buys one ticket.

X = net winnings for 1 ticket bought

Outcome	x	$P(X=x)$
Win $200 basket	200 − 1 = $199	$\frac{1}{1000} = 0.001$
Win $50 dinner gift cert	50 − 1 = $49	$\frac{3}{1000} = 0.003$
Did not WIN	−$1	$\frac{996}{1000} = 0.996$

$\mu = E(X)$
$= 199(0.001) + 49(0.003) - 1(0.996)$
$= \boxed{-\$0.65}$

Example 3: Suppose the yearly premium for a car insurance policy is \$2300 for a customer in a certain category. Statisticians for the insurance company have determined that a person in this category has a 0.007 probability of having an accident that costs the insurance company \$100,000 and a 0.015 probability of having an accident that costs the insurance company \$30,000. What is the expected value of the insurance policy to the customer? To the insurance company?

X = value of policy to the customer

Outcome	Value of X x	$P(X=x)$
Big accident	100 000 − 2300 = \$97 700	0.007
Small accident	30 000 − 2300 = \$27 700	0.015
No accident	− \$2300	1 − 0.015 − 0.007 = 0.978

$\mu = E(X) = 97700\,(0.007) + 27700\,(0.015) - 2300\,(0.978)$

$= \boxed{-\$1150}$ expected value to customer

Expected value to the insurance company is

$\boxed{\$1150}$

Standard deviation of a discrete random variable:

The Standard Deviation of a Discrete Random Variable:

Suppose that a random variable *X* can take on the *n* values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the ~~mean~~ standard deviation of X is

$$\sigma = \sqrt{(x_1-\mu)^2 p_1 + (x_2-\mu)^2 p_2 + ... + (x_n-\mu)^2 p_n}$$
$$= \sqrt{\sum_{i=1}^{n}(x_i-\mu)^2 p_i}$$

Example 4: Calculate the mean and standard deviation of the probability distribution.

x	$P(X = x)$ [sometimes written $P(x)$]
0	0.11
1	0.32
2	0.43
3	0.10
4	0.04

xP

$0(0.11) = 0$

$1(0.32) = 0.32$

$2(0.43) = 0.86$

$3(0.10) = 0.3$

0.16

Find the mean:

$$\mu = E(x) = 0(0.11) + 1(0.32) + 2(0.43) + 3(0.10) + 4(0.04) = \boxed{1.64 = \mu}$$

x	$x - \mu$	$(x-\mu)^2$	$P(x)$	$(x-\mu)^2 P(x)$
0	$0 - 1.64 = -1.64$	$(-1.64)^2 = 2.6896$	0.11	$2.6896(0.11) = 0.295856$
1	$1 - 1.64 = -0.64$	$(-0.64)^2 = 0.4096$	0.32	0.131072
2	$2 - 1.64 = 0.36$	$(0.36)^2 = 0.1296$	0.43	0.055728
3	$3 - 1.64 = 1.36$	$(1.36)^2 = 1.8496$	0.10	0.18496
4	$4 - 1.64 = 2.36$	$(2.36)^2 = 5.5696$	0.04	0.222784
			Sum:	0.8904

Variance:

$\sigma^2 = 0.8904$

std. dev:

$\sigma = \sqrt{0.8904}$

$\boxed{= 0.9436}$

Example 5: Use the frequencies to construct a probability distribution for the random variable X, which represents the number of games bowled by customers at a bowling alley. Calculate the mean and standard deviation of X.

Number of Games	Frequency
1	37
2	45
3	29
4	12
5	4

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Example 5: Use the frequencies to construct a probability distribution for the random variable *X*, which represents the number of games bowled by customers at a bowling alley. Calculate the mean and standard deviation of *X*.

Number of Games	Frequency	Relative Frequency
1	37	37/127
2	45	45/127
3	29	29/127
4	12	12/127
5	4	4/127

$n = 127$

We use the relative frequencies to assign the probabilities in our probability distribution:

x	$P(x)$
1	37/127
2	45/127
3	29/127
4	12/127
5	4/127

(you should check that the probabilities add up to 1)

Mean:

$$\mu = 1\left(\frac{37}{127}\right) + 2\left(\frac{45}{127}\right) + 3\left(\frac{29}{127}\right) + 4\left(\frac{12}{127}\right) + 5\left(\frac{4}{127}\right)$$

$$= \frac{1}{127}\left(1(37) + 2(45) + 3(29) + 4(12) + 5(4)\right) \qquad \left[\text{factor out } \frac{1}{127}\right]$$

$$= \frac{1}{127}(282) = \frac{282}{127} \approx 2.2205 \leftarrow \text{store in calculator, if possible.}$$

$\boxed{\text{The mean number of games is 2.2205.}}$

x	$x-\mu$	$(x-\mu)^2$	$P(x)$	$(x-\mu)^2 p(x)$
1	1-2.2205	$(-1.2205)^2 \approx 1.48955$	37/127	0.433963
2	2-2.2205	$(-0.2205)^2 \approx 0.04861$	45/127	0.017224
3	3-2.2205	$(0.7795)^2 \approx 0.60766$	29/127	0.138757
4	4-2.2205	$(1.7795)^2 \approx 3.16662$	12/127	0.299218
5	5-2.2205	$(2.7795)^2 \approx 7.72577$	4/127	0.243331

$\sigma^2 = \text{sum} = 1.132493 \leftarrow$ Variance

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.132493} \approx \boxed{1.064 \text{ std. dev. of \# of games}}$$