9.2: Critical-Value Approach to Hypothesis Testing

How do we decide whether the null hypothesis is tenable? Or whether there is evidence in favor of the alternative hypothesis? There are two approaches. In both approaches, we set the α-level and state the null and alternative hypotheses before the sample data is analyzed. Then:

Approach #1: *p*-value approach to hypothesis testing (Section 9.3; we'll omit):

First, we calculate the test statistic. If we are interested in testing a hypothesis about the population mean, then the test statistic is the sample mean.

Then we use the test statistic to calculate a *p*-value, often by using technology. The *p*-value, called the *observed significance level*, is the probability of obtaining a sample statistic at least as extreme as that observed in the sample, given that the null hypothesis is true.

A result is said to be *statistically significant* if the *p*-value is less than the predetermined α level.

If the *p*-value is less then or equal to α, we reject the null hypothesis.
If the *p*-value is greater than α, we fail to reject the null hypothesis.

Note: We cannot prove the null hypothesis is true. We never accept the null hypothesis. The closest we can come to accepting the null hypothesis is to conclude that there is not enough evidence to reject it.

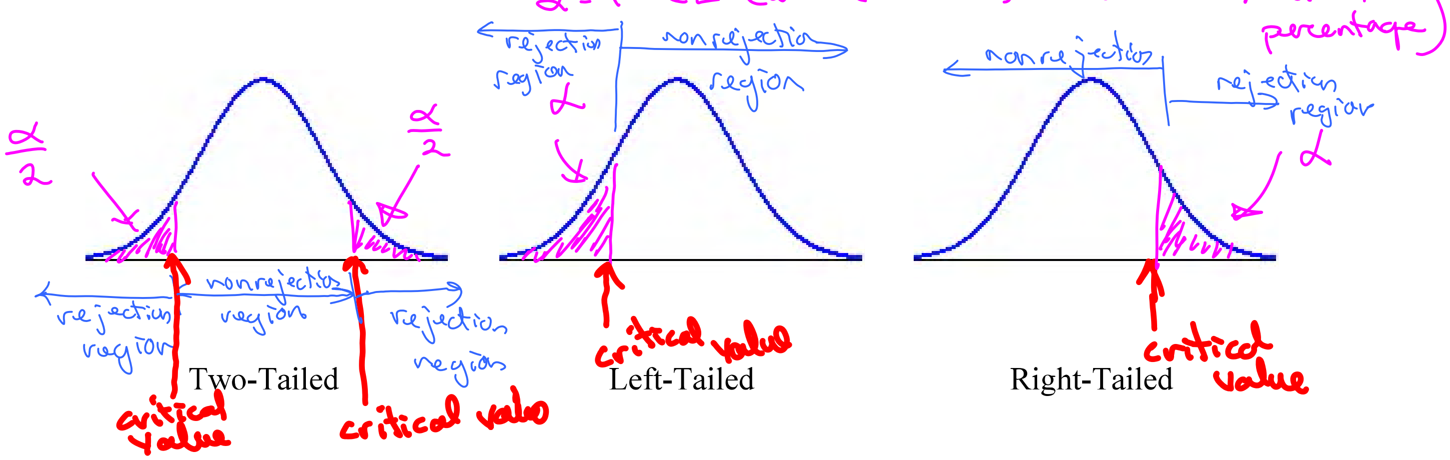
us! **Approach #2: Critical-value approach to hypothesis testing (Section 9.2):**

Determine the rejection region, nonrejection region, and critical value(s).

Rejection region: Set of values for the test statistic that lead to rejection of the null hypothesis.

Nonrejection region: Set of values for the test statistic that do not lead to rejection of the null hypothesis.

Critical value(s): Value(s) of the test statistic that separate the rejection region from the nonrejection region. (The critical values are considered to be included in the rejection region.) We'll use a table to find them. $\alpha = 1 - CL$ (where CL is a decimal, not a percentage)



Calculate the test statistic and compare it to the critical value.

- If the test statistic falls in the rejection region, reject the null hypothesis.
- If the test statistic falls in the nonrejection region, do not reject the null hypothesis.

Obtaining critical values for a one-mean z-test:

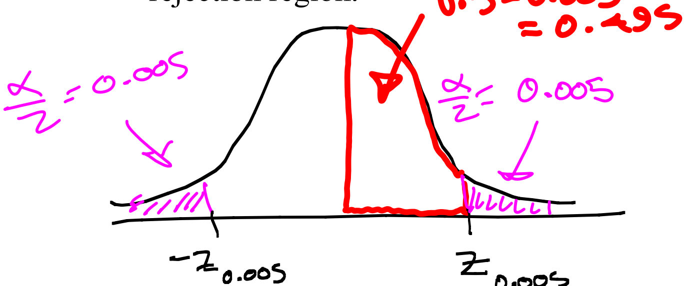
We test a hypothesis about one population mean, in which the null hypothesis H_0 is $\mu \neq \mu_0$, $\mu < \mu_0$, or $\mu > \mu_0$.

The one-mean z-test is used when the population standard deviation is known and the variable under consideration is normally distributed. The test statistic is

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} . \qquad = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$$

benchmark

Example 1: Determine the critical value(s) for a two-tailed z-test with $\alpha = 0.01$. Sketch the rejection region.

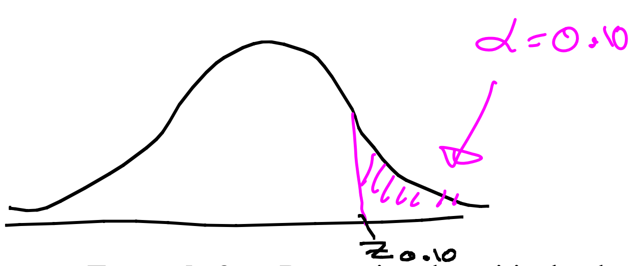


$\alpha = 0.01$

Put $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$ in each tail

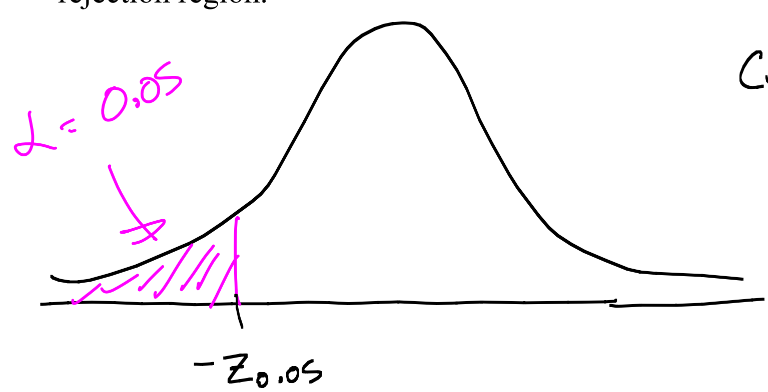
Look up area = 0.495 in normal table.
Closest areas are 0.4949 and 0.4951, corresponding to $z = 2.57$ and $z = 2.58$. So $\boxed{z_{0.005} = 2.575}$ critical value

Example 2: Determine the critical value(s) for a right-tailed z-test with $\alpha = 0.10$. Sketch the rejection region.



From z-values at bottom of t-table, the critical value is $\boxed{z_{0.10} = 1.282}$

Example 3: Determine the critical value(s) for a left-tailed z-test with $\alpha = 0.05$. Sketch the rejection region.



Critical value is

$z_{0.05} = 1.645$