9.5: Hypothesis Tests for One Population Mean When σ is Unknown

In practice, when we are using a sample to make inferences about the population mean, it is rare for us to know the population standard deviation.

Instead, we must use the sample standard deviation, s, as a point estimate of the population standard deviation, σ.

When using s as an estimate for σ, we cannot use a z-test, because $\dfrac{\overline{x}-\mu}{\frac{s}{\sqrt{n}}}$ is not normally distributed.

The *t*-test for one population mean:

When using s as an estimate for σ, we use the Student t-distribution.

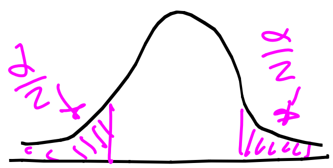
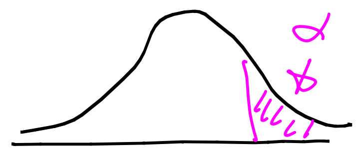
In order to use this procedure, we need to know (or be able to reasonably assume) that the variable of interest follows a normal distribution, or we must have a large sample size ($n \geq 30$).

In addition, the sample should be randomly obtained, observations within the sample must be independent of one another. This means that if we have a sample size that is more than 5% of the population, we should multiply the standard error by a finite population correction factor, $\sqrt{\dfrac{N-n}{n-1}}$. (In this class, I do not anticipate that we will encounter this situation.)

Hypothesis Testing for a Population Mean:

Step 1: Determine the significance level α .

Step 2: Determine the null and alternative hypotheses.

Two-Tailed Test (most common)	Left-Tailed Test (rare)	Right-Tailed Test (rare)
$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$
 Rejection Region	 Rejection Region	 Rejection Region

Note: One tailed tests assume that the scenario not listed ($\mu > \mu_0$ for a left-tailed test or $\mu < \mu_0$ for a right-tailed test) is not possible or is of zero interest.

μ_0 = benchmark.

Step 3: Use your α level and hypotheses, sketch the rejection region.

Step 4: Compute the test statistic $t = \dfrac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$.

Step 5: Use a table (Table IV, on page ~~[illegible]~~) to determine the critical value for t associated with your rejection region.

A-6, A-7

Step 6: Determine whether the value of t calculated from your sample (in Step 3) is in the rejection region.

- If t is in the rejection region, reject the null hypothesis.
- If t is not in the rejection region, do not reject the null hypothesis.

Step 7: State your conclusion.

Example 1: The normal human body temperature is widely accepted to be 98.6° F and can be assumed to follow a normal distribution. A medical researcher wants to know whether a certain geographical community of Native Alaskans' [redacted] a mean body temperature differs from [redacted] 98.6° F. A sample of 20 members of the Native Alaskan geographical community resulted in a mean body temperature of 98.3° F with a standard deviation of 0.7° F. Perform an appropriate hypothesis test at the 95% confidence level.

$H_0: \mu = 98.6°F$

$H_1: \mu \neq 98.6°F$

$\alpha = 1 - 0.95 = 0.05$

Sample info

$n = 20$

$\bar{x} = 98.3°F$

$s = 0.7°F$

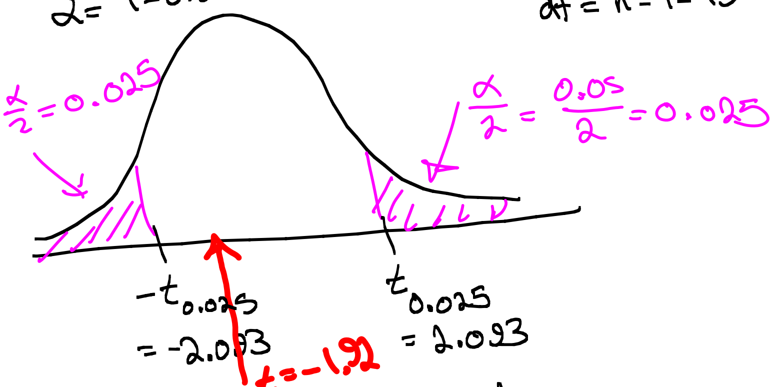
$df = n - 1 = 19$

Find t for our sample

std. error $\frac{s}{\sqrt{n}} = \frac{0.7}{\sqrt{20}} = 0.1565$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{98.3°F - 98.6°F}{0.1565}$$

$$= -1.92$$



From table, for $df = 19$ and tail area $= 0.025$, the critical value of t is 2.093

This t is not in the rejection region, so we $\boxed{\text{do not reject } H_0}$.

$\boxed{\text{This sample does not provide evidence that the pop. of Alaskans' body temperature differs from } 98.6°F}$

Example 2: The average amount of lead in the blood of young children is 2 micrograms per deciliter (mcg/dL). A city has recently changed its water supply, and there have been widespread reports of increased lead levels in the water. A concerned doctor wants to dig into the city's medical records to find out whether the children in the city have blood lead levels above the average level of 2 mcg/dL. In a sample of 35 children, she found a mean lead level of 2.60 mcg/dL with a standard deviation of 1.9 mcg/dL. Perform an appropriate hypothesis test at the 95% confidence level.

one-tailed

$H_0: \mu = 2 \text{ mcg/dL}$

$H_1: \mu > 2 \text{ mcg/dL}$

$\alpha = 1 - 0.95 = 0.05$

Sample info

$n = 35$. Note that $n \geq 30$, so we can assume the sample means are normally distributed

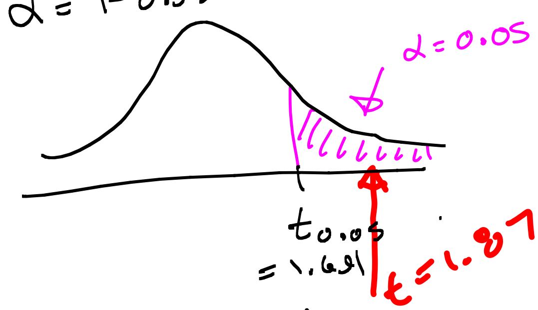
$\bar{x} = 2.60 \text{ mcg/dL}$

$s = 1.9 \text{ mcg/dL}$

Calculate t for our sample:

Standard error: $\frac{s}{\sqrt{n}} = \frac{1.9}{\sqrt{35}} \approx 0.32116$

$$t = \frac{\bar{x} - \text{benchmark}}{\text{std. error}} = \frac{2.60 - 2}{0.32116} = 1.87$$



Find critical value of t.

$df = 35 - 1 = 34$. From table, critical value is $t_{0.05} = 1.691$

$= n - 1$.

$t = 1.87$ is in reject region, so we $\boxed{\text{reject } H_0}$. This sample provides evidence that the pop of kids in this town likely have lead levels above normal.

Example 3: Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. A customer (or a FDA analyst) buys 50 bags of chips, weighs them on a high-accuracy scale, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.

$H_0: \mu = 8$ oz

$H_1: \mu \neq 8$ oz

two-tailed test

$\alpha = 0.10$ (90% confidence level)

Sample info

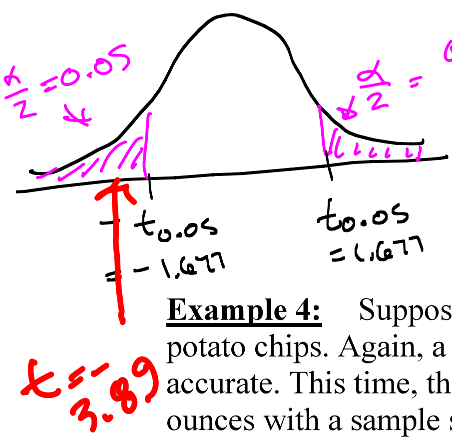
$n = 50$ (note $n \geq 30$)

$\bar{x} = 7.89$ oz

$s = 0.2$ oz

Find critical value: $df = n - 1 = 49$

$t_{0.05} = 1.677$



Calculate t for our sample

std error: $\frac{s}{\sqrt{n}} = \frac{0.2}{\sqrt{50}} \approx 0.02828$

$t = \frac{\bar{x} - 8}{0.02828} \approx -3.89$

Reject H_0. This sample provides evidence that 8 oz is not correct package size.

Example 4: Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. Again, a customer (or a FDA analyst) wonders whether the package size is accurate. This time, the analysist only buys 10 bags of chips, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.