Supplement: Basic Set Theory

Definition: A *set* is a well-defined collection of objects. Each object in a set is called an *element* of that set.

set of all U.S. citizens living in the U.S. currently

Examples of sets:

set of all positive integers
set of students enrolled in this class
set of NBA players

Not sets:

collection ~~set~~ of all tall men
All cute dogs

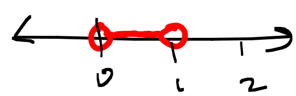
Sets can be finite or infinite.

Examples of finite sets:

NBA players

Examples of infinite sets:

Set of integers
Set of numbers between 0 and 1



Set of possible weights for a human.

Notation:

- We usually use capital letters for sets.
  We usually use lower-case letters for elements of a set.

$\in$: "is an element of" (is a member of)

- $a \in A$ means *a* is an element of the set *A*.
  $a \notin A$ means *a* is not an element of the set *A*.

$a \in A$
$a \notin A$

- The *empty set* is the set with no elements. It is denoted $\varnothing$. This is sometimes called the *null set*.

$\varnothing$

- $S = \{x \mid P(x)\}$ means "*S* is the set of all *x* such that $P(x)$ is true". (called rule notation or set roster notation).

or set builder notation

Example: $S = \{x \mid x \text{ is an even positive integer}\}$ means $S = \{2, 4, 6, 8, \ldots\}$

↑ "such that"

$\{(x,y) \mid x < y\}$

- $n(A)$ means the number of elements in set *A*.

$n(A)$

Definition: We say two sets are *equal* if they have exactly the same elements.

Subsets:

Definition: If each element of a set A is also an element of set B, we say that A is a *subset* of B. This is denoted $A \subseteq B$ or $A \subset B$. If A is not a subset of B, we write $A \not\subset B$.

$A \subseteq B$ or $A \subset B$

Definition: We say A is a *proper subset* of B if $A \subseteq B$ but $A \neq B$. (In other words, every element of A is also an element of B, but B contains at least one element that is not in A.)

Note on notation: Some books use the symbol $\subset$ to indicate a proper subset. Some books use $\subset$ to indicate any subset, proper or not.

Definition: The set of all elements under consideration is called the *universal set*, usually denoted U.

Example: If you're dealing with sets of real numbers, then U is the set of all real numbers. So "Wednesday" would not be an element of U, but 5.7 would be in U.

Example 1: Consider these sets.

$$A = \{1,2,3,4,5,6\}$$
$$B = \{1,2,3,4,5,6,7,8\}$$
$$C = \{1,3,5,2,4,6\}$$

$A \subseteq B$

$C \subseteq B$

$A = C$

Note:

- $\varnothing$ is a subset of every set. (i.e. $\varnothing \subseteq A$ for every set A.)
- Every set is a subset of itself. (i.e. $A \subseteq A$ for every set A.)

Example 2: List all subsets of $\{1,2,3\}$.

$\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\varnothing$, $\{1,2,3\}$

Note: If a set has n elements, how many subsets does it have?

2^n

So set $\{1,2,3\}$ (or set $\{4,5,6\}$) has $2^3 = 8$ subsets

Set operations:

- Union $\cup$: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

  Key word: OR

- Intersection $\cap$: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

  Key word: AND

- Complement A' or A^C or $A^{\sim}$: $A' = \{x \in U \mid x \notin A\}$.

  A' or A^C or $A^{\sim}$

  Key word: NOT

Note: $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.
$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.

<u>Definition</u>: We say that A and B are *disjoint sets* if $A \cap B = \varnothing$.
(not possible to be in both A and B)

<u>Example 3:</u> $U = \{1,2,3,4,5,6,7,8\}$
$H = \{1,3,5,7\}$
$K = \{1,2,3\}$
$J = \{2,4,6,8\}$
$L = \{1,2\}$

$H \cap K = \{1,3\}$
(h intersect K) (must be in both H and K

$H \cup K = \{1,3,5,7,2\} = \{1,2,3,5,7\}$
(h union K

$J \cup K = \{1,2,3,4,6,8\}$
$J \cap K = \{2\}$

K complement

$K' = \{4,5,6,7,8\}$, $H' = \{2,4,6,8\}$
our book writes Not K

Venn Diagrams: These help us visualize set relationships and operations.

<u>Example 4:</u> Draw Venn diagrams for $A \cup B$, $A \cap B$, A^C, B^C, $(A \cap B)^C$, and $(A \cup B)^C$.