1.2: Finding Limits Graphically and Numerically

Limit of a function:

Definition of a Limit:

$$\lim_{x \to a} f(x) = L$$

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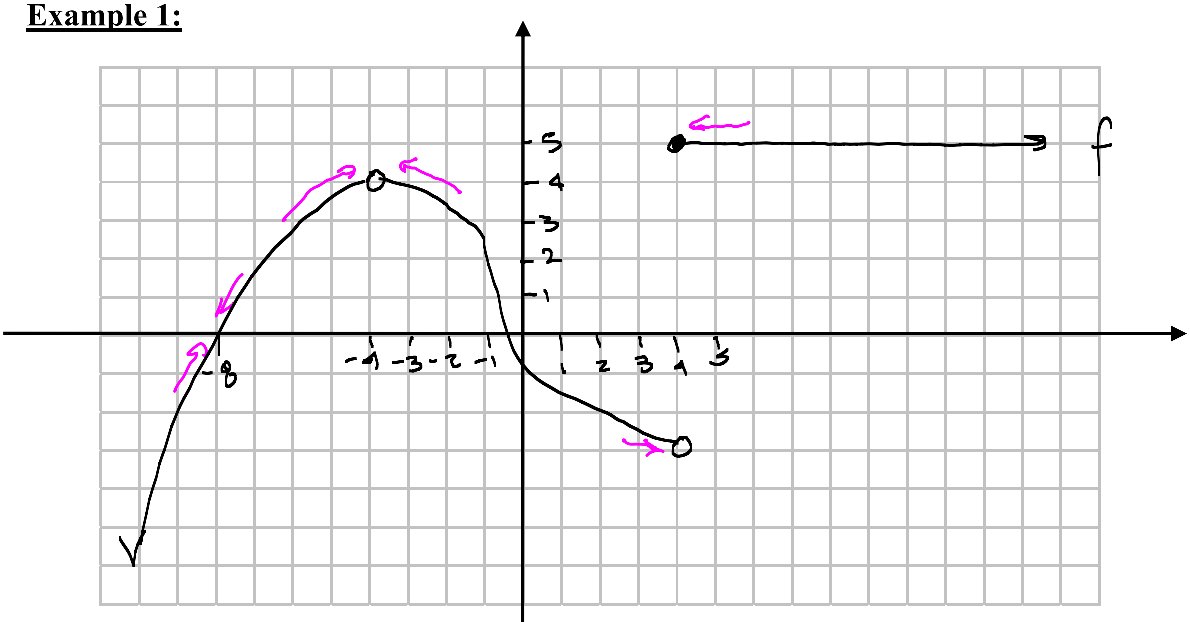
The statement above means that we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a but not equal to a.

We read this as "the limit of $f(x)$, as x approaches a, is equal to L."

Alternative notation: $f(x) \to L$ as $x \to a$. ($f(x)$ approaches L as x approaches a)

Finding limits from a graph:

Example 1:



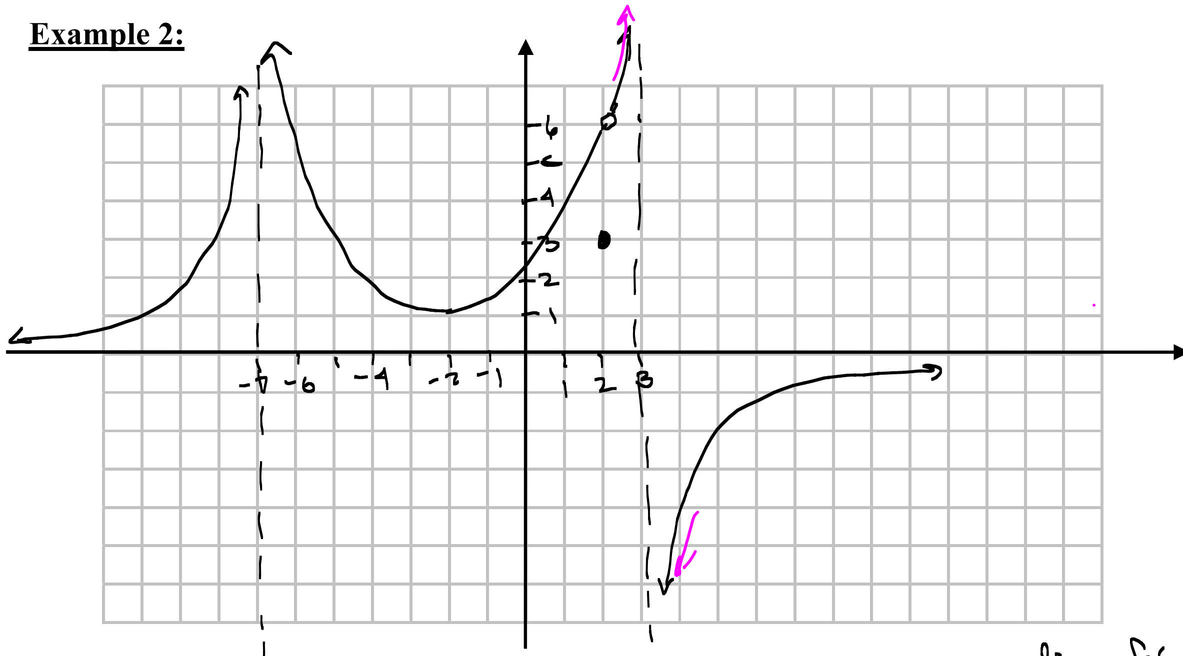
$\lim_{x \to -4} f(x) = \underline{\ 4\ }$

$\lim_{x \to -8} f(x) = \underline{\ 0\ }$

$\lim_{x \to 4} f(x)$ Does not exist

(because it approaches different y-values from the left and right)

Example 2:



$\lim_{x\to 3} f(x)$ Does not exist

$\lim_{x\to 2} f(x) = 6$

Note: $f(2) = 3$

$\lim_{x\to -7} f(x) = \infty$

so, $\boxed{\lim_{x\to -7} f(x) \text{ does not exist}}$

$\lim_{x\to -8} f(x) = 3$

Note: $f(-8) = 3$

Example 3: Graph the function. Use the graph to determine $\lim_{x\to 2} f(x)$ and $\lim_{x\to -1} f(x)$.

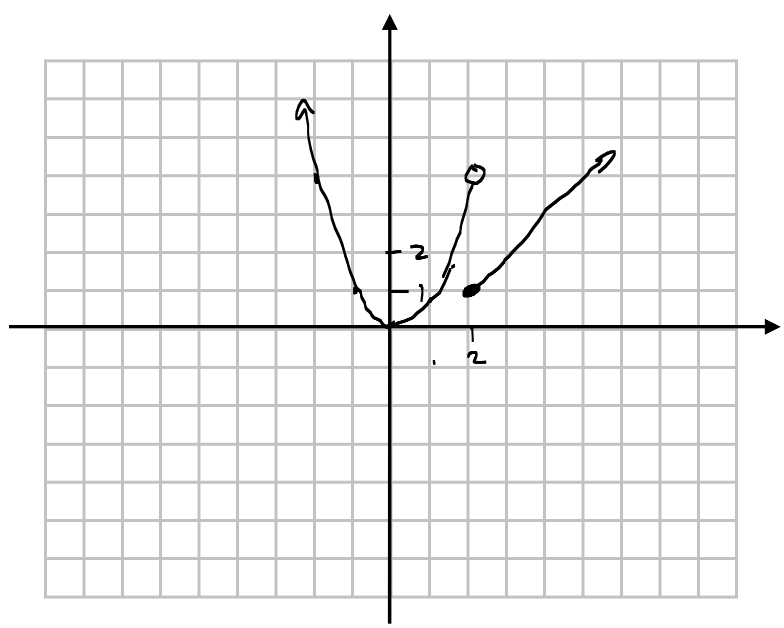
$$f(x) = \begin{cases} x-1 & \text{if } x \geq 2 \\ x^2 & \text{if } x < 2 \end{cases}$$

right of 2: $y = 1x - 1$

$m = 1 = \frac{1}{1}$ = slope

$b = -1 \Rightarrow$ y-intercept

left of 2: $y = x^2$



$\lim_{x\to 2} f(x)$ does not exist

$\lim_{x\to -1} f(x) = 1$

Finding limits numerically:

Example 4: For the function $f(x) = \dfrac{x-5}{x^2-25}$, make a table of function values corresponding to values of x near 5.

Note: $f(5)$ is undefined (zero denominator)

Use the table to estimate the value of $\lim\limits_{x \to 5} \dfrac{x-5}{x^2-25}$.

x	$f(x)$
4	$\frac{4-5}{4^2-25} = \frac{-1}{-9} = 0.1111$
4.5	$\frac{4.5-5}{(4.5)^2-25} = 0.10526$
4.8	
4.9	
4.99	
4.999	
5.2	
5.1	
5.01	
5.001	
⋮	

Done in class on calculator.

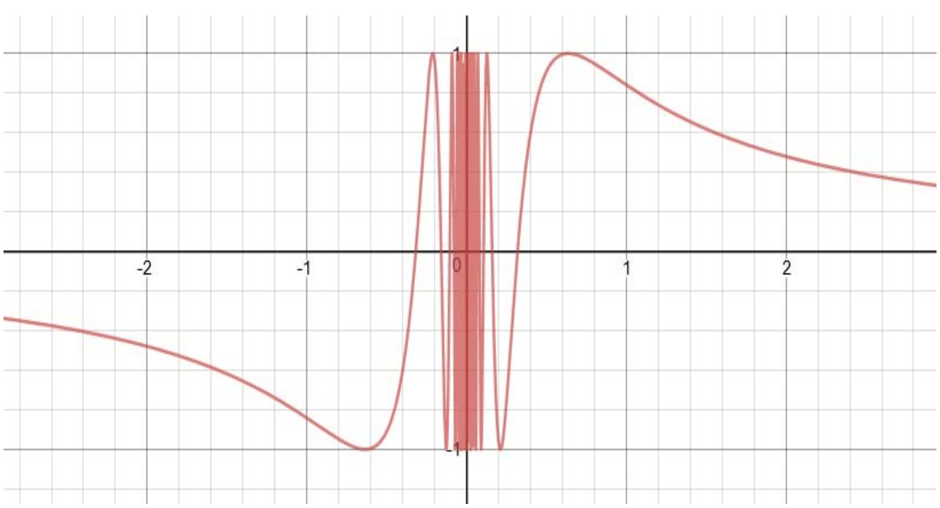
From table, it appears that

$$\lim_{x \to 5} \frac{x-5}{x^2-25} = \boxed{0.1}$$

Example 5: Make a table of values and use it to estimate $\lim\limits_{x \to 0} \sin\left(\dfrac{1}{x}\right)$.

$\lim\limits_{x \to 0} \sin\left(\dfrac{1}{x}\right)$ $\boxed{\text{does not exist}}$

Done on calculators, board, desmos.com



<u>Common reasons</u> $\lim_{x \to c} f(x)$ <u>may not exist:</u>

1. $f(x)$ approaches a different value when approached from the left of c, compared to when approached from the right of c.

2. $f(x)$ increases or decreases without bound as x approaches c.

3. $f(x)$ oscillates between two values as x approaches c.

The formal (epsilon-delta) definition of a limit:

<u>Definition</u>:

Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then

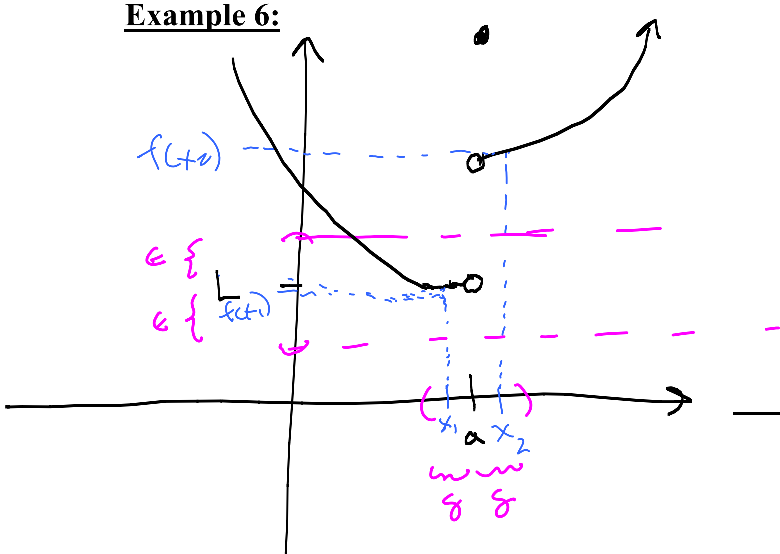
ϵ or ε : epsilon
δ : delta (lower-case)

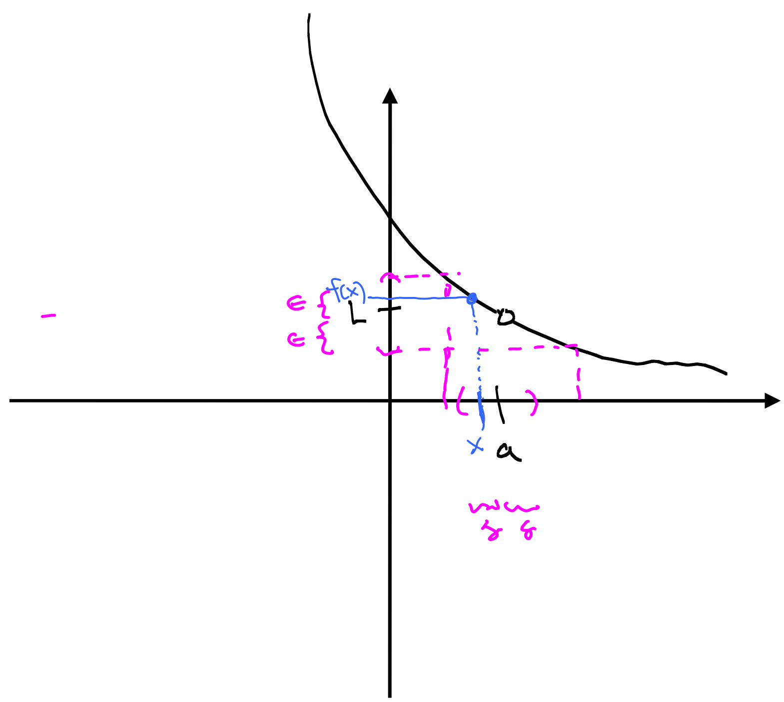
$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$, there is a number $\delta > 0$ such that

$$|f(x) - L| < \varepsilon \text{ whenever } 0 < |x - a| < \delta .$$

<u>Example 6:</u>



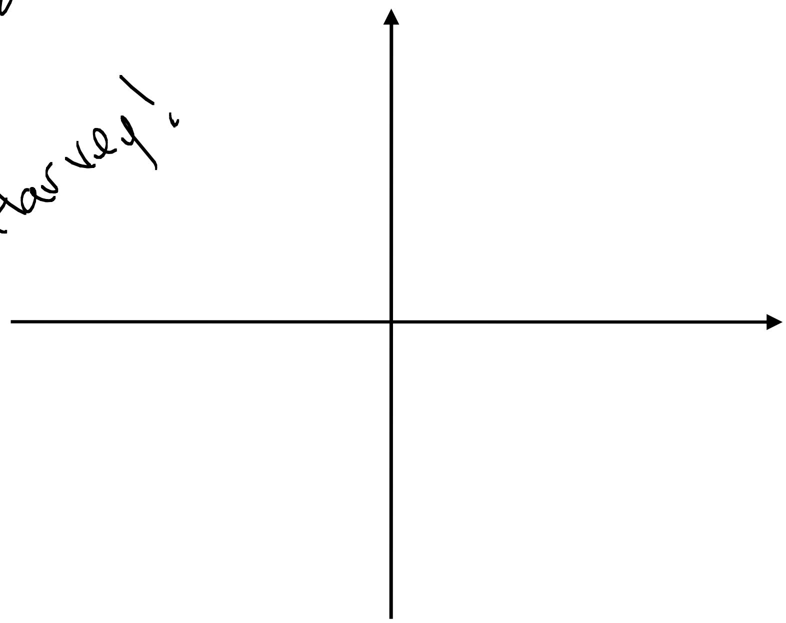


Here, $\lim_{x \to a} f(x) = L$ is <u>not</u> <u>true</u>

because we found an ϵ for which no δ will work

Example 7:

Omit the rest of the ϵ-δ stuff. Thanks, Harvey!



Example 8: How close to 3 must we take x so that $6x-7$ is within 0.1 of 11?

Example 9: How close to 4 must we take x so that x^2-2 is within 0.01 of 14?

Example 10: Prove that $\lim_{x \to 4}(2x-5)=3$ using the definition of a limit.

Example 11: Prove that $\lim_{x \to -3}(5x+1)=-14$ using the definition of a limit.

<u>Example 12:</u> Prove that $\lim_{x \to 3} x^2 = 9$ using the definition of a limit.

<u>Example 13:</u> Prove that $\lim_{x \to 2} (x^2 - x + 6) = 8$ using the definition of a limit.