Note Title 2/5/2018

From RR 4:

a) $3(x-4) = -12-x$

$3x-12 = -12-x$
$+x \qquad\qquad +x$

$4x-12 = -12$
$+12 \qquad +12$

$4x = 0$

$\frac{4x}{4} = \frac{0}{4}$

$x = 0$

Sol'n set: $\boxed{\{0\}}$

Check: $3(0-4) = -12-0$

$3(-4) = -12$

$-12 = -12$

✓ True!

b) $4(5x-2) = 2(10x-1)-6$

$20x-8 = 20x-2-6$

$20x-8 = 20x-8$
$+8 \qquad\qquad +8$

$20x = 20x$
$-20x \qquad -20x$

$0 = 0$

OR

$20x-8 = 20x-8$
$-20x \qquad -20x$

$-8 = -8$

Sol'n Set: $\boxed{\text{All Real Numbers}}$

2.2.3

## Summary: 3 types of equations

(3 situations we need to recognize when solving an equation).

1) Conditional equation: true for some (or one) value(s) of the variable, false for all other values (most common) ... for linear eqns, we get one solution.

2) Contradiction: equation is false for all values of the variable

3) Identity: equation is true for all values of the variable.

* If the variable doesn't disappear, but leaves you with $x =$ some number, it's a conditional equation.

* If the variable disappears and leaves you with a false statement, the eqn is a contradiction and has no solution.
(ex: $0 = 5$)

* If the variable disappears and leaves you with a true statement, the equation is an identity and the solution set is all real numbers.
(Ex: $0 = 0$
$-8 = -8$)

# 2.3: Solving Equations: Clearing Fractions and Decimals

(an alternative approach to handling fractions)

To clear fractions, multiply both sides by the least common denominator (LCD) of all the fractions.

Example: Solve.

$$\frac{1}{5}y + 3 = \frac{7}{6}$$

LCD: 30

$$\frac{30}{1}\left(\frac{1}{5}y + 3\right) = \left(\frac{7}{6}\right)\frac{30}{1}$$

$$\frac{\overset{6}{\cancel{30}}}{1}\left(\frac{1}{\cancel{5}}y\right) + 30(3) = \frac{7}{\cancel{6}}\left(\frac{\overset{5}{\cancel{30}}}{1}\right)$$

$$6y + 90 = 35$$

$$-90 \quad -90$$

$$6y = -55$$

$$\frac{6y}{6} = \frac{-55}{6}$$

$$y = -\frac{55}{6}$$

Sol'n Set: $\boxed{\left\{-\frac{55}{6}\right\}}$

Ex: Solve

$$\frac{3}{4}x - \frac{2}{3} + \frac{1}{6} = 4x - \frac{5}{2}$$

method 1: Clearing Fractions & Decimals

LCD: 12

$$\frac{\cancel{12}^{3}}{1}\left(\frac{3}{\cancel{4}}x\right) - \frac{2}{\cancel{3}}\left(\frac{\cancel{12}^{4}}{1}\right) + \frac{1}{\cancel{6}}\left(\frac{\cancel{12}^{2}}{1}\right) = 12(4x) - \frac{5}{\cancel{2}}\left(\frac{\cancel{12}^{6}}{1}\right)$$

$$9x - 8 + 2 = 48x - 30$$

$$9x - 6 = 48x - 30$$

$$9x - 48x - 6 = -30$$

$$9x - 48x = -30 + 6$$

$$-39x = -24$$

$$\frac{-39x}{-39} = \frac{-24}{-39}$$

$$x = \frac{24 \div 3}{39 \div 3}$$

$$x = \frac{8}{13}$$

$$\boxed{\left\{\frac{8}{13}\right\}}$$

method 2 combining terms before clearing out fractions

$$\frac{3}{4}x - \frac{2}{3} + \frac{1}{6} = 4x - \frac{5}{2}$$

$$\frac{3}{4}x - 4x = \frac{2}{3} - \frac{1}{6} - \frac{5}{2}$$

$$\frac{3}{4}x - \frac{4}{1}x\left(\frac{4}{4}\right) = \frac{2}{3}\left(\frac{2}{2}\right) - \frac{1}{6} - \frac{5}{2}\left(\frac{3}{3}\right)$$

$$\frac{3}{4}x - \frac{16}{4}x = \frac{4}{6} - \frac{1}{6} - \frac{15}{6}$$

$$-\frac{13}{4}x = -\frac{12}{6}$$

$$\frac{-\frac{13}{4}x}{-\frac{13}{4}} = \frac{-\frac{12}{6}}{-\frac{13}{4}}$$

$$-\frac{4}{13}\left(-\frac{13}{4}x\right) = -\frac{\cancel{12}^{2}}{\cancel{6}}\left(-\frac{4}{13}\right)$$

$$x = \frac{8}{13}$$

$$\boxed{\left\{\frac{8}{13}\right\}}$$

To clear decimals, multiply both sides by a power of 10.

Ex: Solve. $0.75(m-2) + 0.25m = 0.5$

multiply both sides by 100:

$$(100)\,0.75(m-2) + (100)\,0.25m = 0.50(100)$$

$$75(m-2) + 25m = 50$$

$$75m - 150 + 25m = 50$$

$$100m - 150 = 50$$

$$100m - 150 = 50$$
$$+150 \quad +150$$

$$100m = 200$$

$$\frac{100m}{100} = \frac{200}{100}$$

$$m = 2 \quad \boxed{\{2\}}$$

# 2.8: Linear Inequalities

$<$ means "is less than"

$>$ means "is greater than"

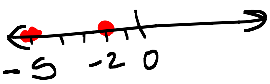
$\leq$ means "is less than or equal to"

$\geq$ means "is greater than or equal to"

**Note:** $\geq$ and $\geqslant$ are equivalent

$\leq$ and $\leqslant$ are equivalent

**True/false:**

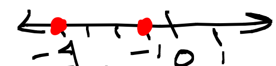



$2 < 5$ True

$-5 > -2$ False

$2 < 2$ False

$2 \leq 2$ True




$-1 < -4$ False

$6 > -12$ True

$8 \geqslant 2$ True

$8 = 2$ False

A linear inequality (in 1 variable) is a relationship that can be written in the form

$ax + b < c$ $\quad$ $ax + b > c$

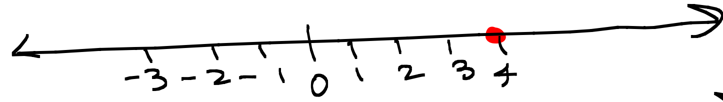
$ax + b \leq c$ $\quad$ $ax + b \geqslant c$

**Ex:** $2x - 3 < 6$ , $2(5 - x) \geqslant x + 3$

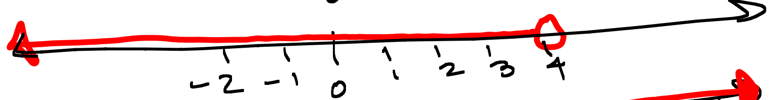
($x$ to $1^{st}$ power only)

We can use a number line to represent the solution set of an inequality:

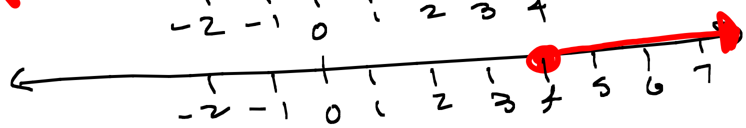
Ex: $x = 4$




$x < 4$




$x \geq 4$



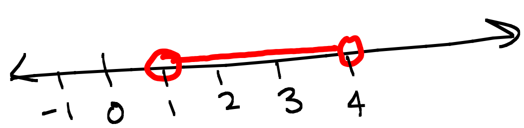

Set-builder notation
(set roster notation)

Inequality: $x < 4$ set-builder notation: $\{x \mid x < 4\}$

"such that"

The set of all numbers x such that $x < 4$.

Interval notation: describe the interval using its left and right boundaries

| Set-builder notation | Graph | Interval notation |
| --- | --- | --- |
| |   | $(1, 4)$ |
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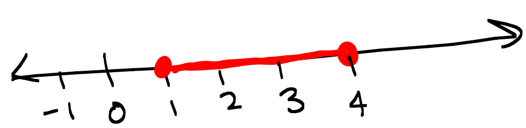
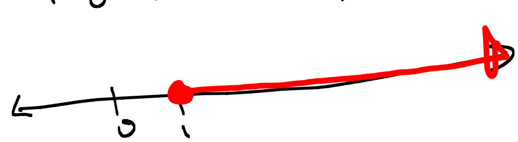
| |   | $[1, 4]$ |
| |   | $[1, \infty)$ |