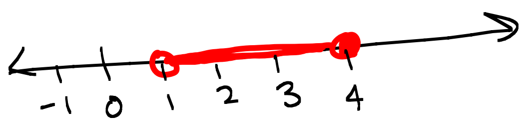
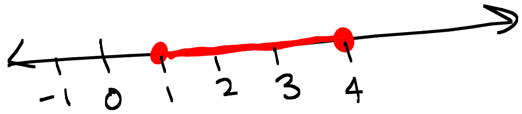
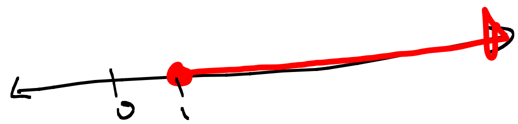
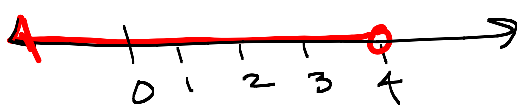
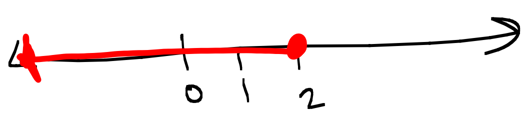
2.8: Linear Inequalities (cont'd)

2.8.3

Note Title 2/7/2018

Set-builder notation (set roster notation)	Graph	Interval notation
$\{x \mid 1 < x < 4\}$		$(1,4)$
$\{x \mid 1 < x \leq 4\}$		$(1,4]$
$\{x \mid 1 \leq x \leq 4\}$		$[1,4]$
$\{x \mid x \geq 1\}$	$-\infty$  ∞	$[1,\infty)$
$\{x \mid x < 4\}$	$-\infty$  ∞	$(-\infty,4)$
$\{x \mid x \leq 2\}$	$-\infty$  ∞	$(-\infty,2]$
$\{x \mid x > -3\}$		$(-3,\infty)$

A <u>linear inequality</u> is an inequality that can be written as

$ax+b<c$ $\quad ax+b\leq c$

$ax+b>c$ $\quad ax+b\geq c$ (or similar)

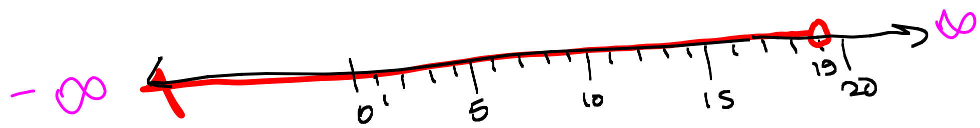
Additive Property of Inequality

If $a < b$, then $a + c < b + c$.

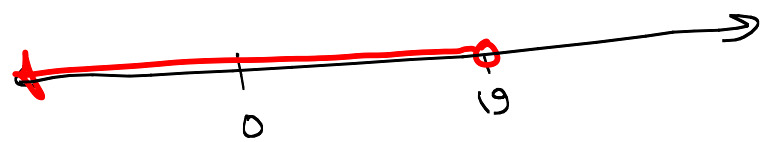
This means we can add (or subtract) the same quantity on both sides.

Example: Solve. $x - 7 < 12$

$$x - 7 < 12$$
$$+7 \quad +7$$
$$x < 19$$



OR



Interval Notation: $(-\infty, 19)$

Set roster: $\{x \mid x < 19\}$

Variable should come first in set-builder notation
(write $x < 19$ not $19 > x$)

Multiplicative Property of Inequality

If $a < b$ and $c > 0$, then $ac < bc$.

If $a < b$ and $c < 0$, then $ac > bc$.

This means:

* we can multiply (or divide) both sides by the same positive number.
* we can multiply (or divide) both sides by the same negative number if we reverse the inequality sign.

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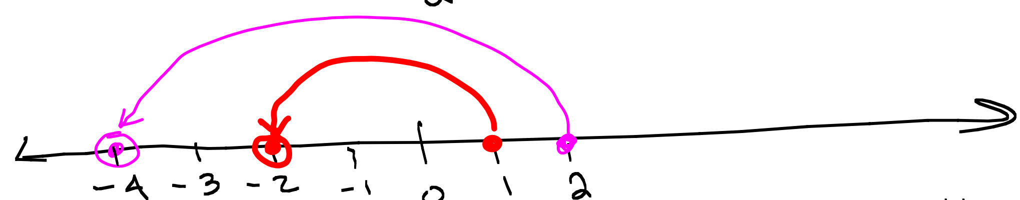
Why must we reverse the inequality sign when multiplying/dividing by a negative?

$1 < 2$ True

multiply both sides by 2: $1(2) < 2(2)$
$2 < 4$ still true!

Start with $1 < 2$ again. ← true.

multiply both sides by -2: $1(-2) < 2(-2)$
$-2 < -4$ False!



(multiplying by a negative reverses the order)

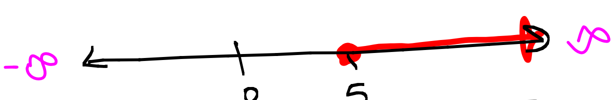
To force the inequality to be equivalent to the original (still true), we reverse the inequality sign when multiplying/dividing by a negative.

Ex: Solve.

$4x \geq 20$

$\frac{4x}{4} \geq \frac{20}{4}$

$x \geq 5$



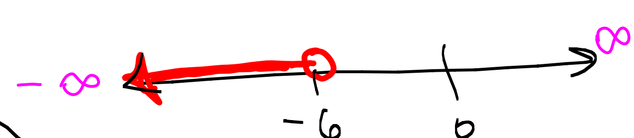
$[5, \infty)$

$\{x \mid x \geq 5\}$

Ex: Solve. $-3x > 18$

Reverse the inequality sign! → $\frac{-3x}{-3} < \frac{18}{-3}$

$x < -6$



$(-\infty, -6)$

$\{x \mid x < -6\}$

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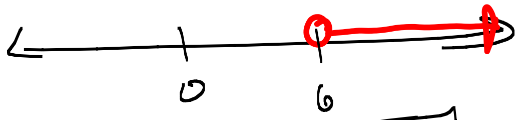
Ex: $-3x < -18$

Reverse the inequality sign $\rightarrow$ $\dfrac{-3x}{-3} > \dfrac{-18}{-3}$

$x > 6$

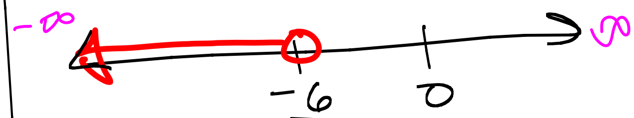


$\boxed{(6, \infty)}$

Ex: $3x < -18$

$\dfrac{3x}{3} < \dfrac{-18}{3}$

$x < -6$



$\boxed{(-\infty, -6)}$

Ex: $2x + 2 \leq 6x + 32$ Solve.

$-2x \qquad -2x$

$2 \leq 4x + 32$

$-32 \qquad -32$

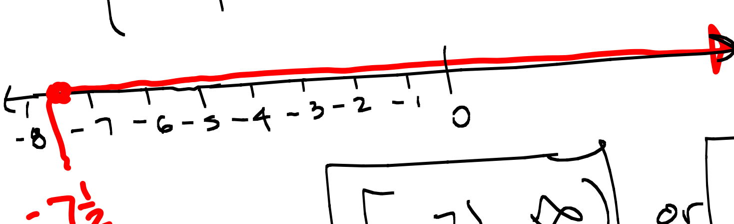
$-30 \leq 4x$

$\dfrac{-30}{4} \leq \dfrac{4x}{4}$

$-\dfrac{15}{2} \leq x$

$x \geq -\dfrac{15}{2}$

$\{x \mid x \geq -7\tfrac{1}{2}\}$



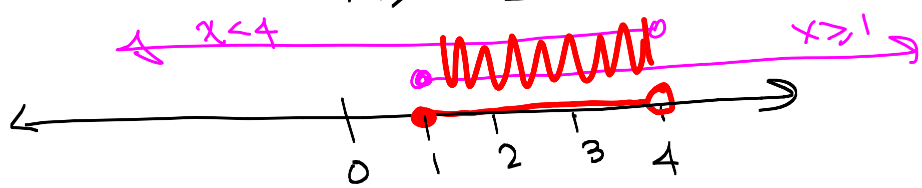
$\boxed{[-7\tfrac{1}{2}, \infty)}$ or $\boxed{[-\tfrac{15}{2}, \infty)}$

Combined inequalities:

The statement $1 \le x < 4$

means $1 \le x$ and $x < 4$

$x \geq 1$ and $x < 4$



Example:

$$-7 \le 2x+5 < 3$$
$$-5 \quad -5 \quad -5$$
$$-12 \le 2x < -2$$
$$\frac{-12}{2} \le \frac{2x}{2} < \frac{-2}{2}$$
$$-6 \le x < -1$$

$$\boxed{[-6,-1)}$$

$$\boxed{\{x \mid -6 \le x < -1\}}$$

Ex: Solve.

$$-4 < 5(3-x) \le 1$$
$$-4 < 15-5x \le 1$$
$$-15 \quad -15 \quad -15$$
$$-19 < -5x \le -14$$
$$\frac{-19}{-5} > \frac{-5x}{-5} \geq \frac{-14}{-5} \leftarrow \text{Reverse both inequality signs!}$$
$$\frac{19}{5} > x \geq \frac{14}{5}$$
$$\frac{14}{5} \le x < \frac{19}{5} \Rightarrow \left\{x \mid 2\tfrac{4}{5} \le x < 3\tfrac{4}{5}\right\}$$