

2.4: Applications of Linear Equations (cont'd)

2.4.2

Note Title

2/19/2018

Example

One number is six more than twice another.
 If their sum is decreased by 10, the result
is 23. Find the numbers.

$$\text{one number} = 2x + 6$$

$$\text{another number} = x$$

one number $\xrightarrow[\text{to}]{\text{compare}} \begin{matrix} \text{another} \\ \# \\ x \end{matrix}$

$$\begin{aligned}
 & (\text{one number}) + (\text{another number}) - 10 = 23 \\
 & \text{sum} \\
 & (2x+6) + (x) - 10 = 23 \\
 & 2x + 6 + x - 10 = 23 \\
 & 3x - 4 = 23 \\
 & + 4 \quad \quad \quad + 4 \\
 & 3x = 27 \\
 & \frac{3x}{3} = \frac{27}{3} \\
 & x = 9
 \end{aligned}$$

$$\text{another number: } x = 9$$

$$\text{one number: } 2x + 6$$

$$\begin{aligned}
 & \text{substitute } x = 9 \Rightarrow 2(9) + 6 \\
 & \text{"implies" } = 18 + 6 \\
 & \text{"leads to" } = 24
 \end{aligned}$$

Write answer in 9
complete sentence:

The numbers are 9 and 24.

Check it!

Ex: Five times the sum of a x number
and seven is thirty. Find the number.

2.4.3

The number: x

$$\begin{array}{rcl} 5(x+7) & = & 30 \\ \text{sum} & & \\ 5x + 35 & = & 30 \\ -35 & & -35 \\ 5x & = & -5 \\ \frac{5x}{5} & = & \frac{-5}{5} \\ x & = & -1 \end{array}$$

The number
is -1 .

Check it: Sum: number + 7
 $\begin{array}{r} -1 + 7 \\ \hline 6 \end{array}$

5 times sum: $5(6)$
 $= 30$ ✓ sentence checks.

Consecutive Integer Problems

(2.4.f)

Example: Find four consecutive integers whose sum is 74.

1st integer: x

2nd integer: $x+1$

3rd integer: $x+2$

4th integer: $x+3$

$$\text{sum} = 74$$

$$(x) + (x+1) + (x+2) + (x+3) = 74$$

$$x + x+1 + x+2 + x+3 = 74$$

$$4x + 6 = 74$$

$$4x = 68$$

$$\frac{4x}{4} = \frac{68}{4}$$

$$x = 17$$

$$\begin{array}{r} 17 \\ 4 \overline{) 68} \\ -4 \\ \hline 28 \\ -28 \\ \hline 0 \end{array}$$

$$1^{\text{st}} \text{ integer: } x = 17$$

$$2^{\text{nd}} \text{ integer: } x+1$$

$$x = 17 \Rightarrow 17+1 = 18$$

$$3^{\text{rd}} \text{ integer: } \frac{x+2}{17+2} = 19$$

$$4^{\text{th}}: \quad \frac{x+3}{17+3} = 20$$

The integers are
17, 18, 19, 20.

Check it: integers? Yes

consecutive? Yes

Is their sum 74?

? 17

. 18

. 19

$$\begin{array}{r} + 20 \\ \hline 74 \end{array}$$

✓ OK!

Ex. Find 3 consecutive even integers whose sum is 120.

2.4.5

1st integer: x

2nd integer: $x+2$

3rd integer: $x+4$

$$(x) + (x+2) + (x+4) = 120$$

$$x + x+2 + x+4 = 120$$

$$3x + 6 = 120$$

-6

$$3x = 114$$

$$\frac{3x}{3} = \frac{114}{3}$$

$$x = 38$$

1st integer: $x = 38$

$$\begin{aligned} \text{2}^{\text{nd}} \text{ integer: } & x+2 \\ & = 38+2 = 40 \end{aligned}$$

$$\begin{aligned} \text{3}^{\text{rd}}: & x+4 \\ & = 38+4 = 42 \end{aligned}$$

The even integers are 38, 40, and 42.

check: integers? Yes

consecutive evens? Yes

3 of them? Yes

Is their sum 120? Yes ✓

$$38 + 40 + 42$$

$$= 120$$

Note: consecutive integers:

$$\begin{array}{l} x \\ x+1 \end{array}$$

$$x+2$$

$$x+3$$

$x+4$ etc

Consecutive even integers:

$$\begin{array}{l} x \\ x+2 \\ x+4 \\ x+6 \text{ etc} \end{array}$$

Consecutive odd integers:

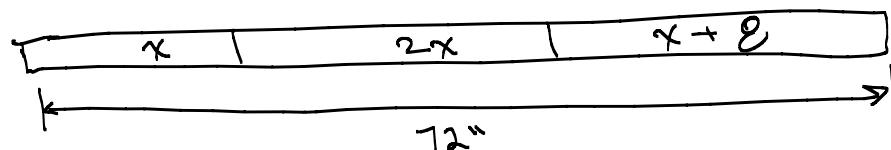
$$\begin{array}{l} x \\ x+2 \\ x+4 \\ x+6 \text{ etc} \end{array}$$

2.4.4

Ex. A 72-inch board is cut into 3 pieces.
 The second piece is twice as long as the first piece.
 The third piece is 8 inches longer than the first piece.
 How long are the pieces?

$$\begin{aligned} \text{length of 1st piece: } &x \\ \text{length of 2nd piece: } &2x \\ \text{length of 3rd piece: } &x+8 \end{aligned}$$

2nd $\xrightarrow{\text{compared to}}$ 1st
 3rd $\xrightarrow{\text{compared to}}$ ~~x~~



$$\begin{aligned} (x) + (2x) + (x+8) &= 72 \\ x + 2x + x + 8 &= 72 \\ 4x + 8 &= 72 \\ 4x &= 72 - 8 \\ 4x &= 64 \\ \frac{4x}{4} &= \frac{64}{4} \\ x &= 16 \end{aligned}$$

1st piece: $x = 16$

2nd piece: $2x$

$$x = 16 \Rightarrow 2(16) = 32$$

3rd piece: $x+8$

$$x = 16 \Rightarrow 16 + 8 = 24$$

The pieces are 16", 32" and 24".

Check it!

2.6: More applications of Linear Application

2.6.1

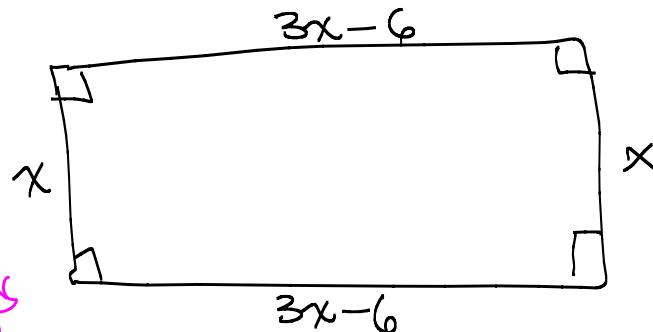
Ex: The perimeter of a rectangle is 100 ft.

The length is 6 ft less than three times the width. Find the length and width.

$$\text{length: } 3x - 6$$

$$\text{width: } x$$

length $\xrightarrow{\text{compare to}}$ width x



Perimeter = sum of all side lengths

$$x + (3x - 6) + x + (3x - 6) = 100$$

$$x + 3x - 6 + x + 3x - 6 = 100$$

$$8x - 12$$

+12

or

$$2(\text{length}) + 2(\text{width}) = \text{Perimeter}$$

$$2(3x - 6) + 2(x) = 100$$

$$= 100$$

$$+12$$

$$8x = 112$$

$$\frac{8x}{8} = \frac{112}{8}$$

$$x = 14$$

$$\text{width: } x = 14$$

$$\text{length: } 3x - 6$$

$$\begin{aligned} x = 14 &\Rightarrow 3(14) - 6 \\ &= 42 - 6 \\ &= 36 \end{aligned}$$

The width is 14 ft
and the length is 36 ft.

5.1: Multiplying and Dividing Expressions with Common Bases

5.1.1

Recall: Powers of positive and negative numbers

$$-3^2 = -(3)(3) = -9$$

$$(-3)^2 = (-3)(-3) = 9$$

$$(-2)^6 = \underbrace{(-2)(-2)}_{+} \underbrace{(-2)(-2)}_{+} \underbrace{(-2)(-2)}_{+} = 4 \cdot 4 \cdot 4 = 16 \cdot 4 = \boxed{64}$$

$$(-2)^5 = \underbrace{(-2)(-2)}_{+} \underbrace{(-2)(-2)}_{+} \underbrace{(-2)}_{-} = 1(4)(-2) = 16(-2) = \boxed{-32}$$

$$-2^6 = -(2)(2)(2)(2)(2)(2) = \boxed{-64}$$

$$-2^5 = -(2)(2)(2)(2)(2) = \boxed{-32}$$

Important:

- * If you raise a negative number to an even exponent, you get a positive number.
- * If you raise a negative number to an odd exponent, you get a negative number.

Know these powers:

5.1.2

In calculator:
look for
 x^y or y^x
key

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

$$2^8 = 256$$

$$2^9 = 512$$

$$\underline{2^{10} = 1024}$$

$$10^2 = 100$$

$$10^3 = 1000$$

$$10^4 = 10000$$

$$10^5 = 100\,000$$

etc

$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

$$3^5 = 243$$

$$3^6 = 729$$

$$4^2 = 16$$

$$4^3 = 64$$

$$4^4 = 256$$

$$9^2 = 81$$

$$9^3 = 729$$

$$5^2 = 25$$

$$5^3 = 125$$

$$5^4 = 625$$

$$6^2 = 36$$

$$6^3 = 216$$

$$7^2 = 49$$

$$7^3 = 343$$

$$8^2 = 64$$

$$8^3 = 512$$