

5.1: Exponents (cont'd)

5.1.3

Note Title

2/21/2018

Properties (Laws) of Exponents

$$1) x^a x^b = x^{a+b}$$

$$\text{Ex 1: } x^2 x^3 = (xx)(xxx) = x^5$$

$$2) (x^a)^b = x^{ab}$$

$$\text{Ex 2: } (x^2)^3 = (xx)^3 = (xx)(xx)(xx) = x^6$$

$$3) \frac{x^a}{x^b} = x^{a-b}$$

$$\text{Ex 3: } \frac{x^7}{x^3} = \frac{\cancel{xxx} \cancel{xxx} x}{\cancel{xxx}} = \frac{x^4}{1} = x^4$$

$$4) (xy)^a = x^a y^a$$

$$\text{Ex 4: } (xy)^3 = (xy)(xy)(xy) = xxx yyy = x^3 y^3$$

$$5) \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

$$\text{Ex 5: } \left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{xxx}{yyy} = \frac{x^3}{y^3}$$

Examples: Simplify.

$$\text{Ex: } \left(-\frac{2}{3}\right)^4 = + \frac{2^4}{3^4} = \boxed{\frac{16}{81}} \leftarrow \text{Prop. \# 5}$$

$$\text{Ex: } \frac{3^{15}}{3^{13}} = 3^{15-13} = \frac{3^2}{1} = \boxed{9} \leftarrow \text{Prop. \# 3}$$

$$\text{Ex: } \frac{2^5}{3^4} = \boxed{\frac{32}{81}}$$

$$\text{Ex: } 2x^3 x^5 = 2x^8$$

$\leftarrow$ Property #1

Ex. $(6a^4)(-3a^2) = 6(-3)a^4a^2$ (5.1.4)
 $= \boxed{-18a^6}$ $\leftarrow$ Prop. #1

Ex. $(-2c^2d^3)(6c^3d)(d) = \boxed{-12c^5d^5}$ Ex. $\frac{a^3a^{11}}{a^4a^5} = \frac{a^{14}}{a^9}$
 $= a^{14-9} = \boxed{a^5}$
Prop #3 $\rightarrow$ Prop. #1 $\downarrow$

Ex. $\frac{15m^5n^{12}}{3m^4n^7} = \frac{\cancel{15}^5 m^{5-4} n^{12-7}}{\cancel{3}_1} = \frac{5mn^5}{1} = \boxed{5mn^5}$

5.2: More Properties of Exponents

5.2.1

Here, we practice with

Property #2 $(x^a)^b = x^{ab}$

"Power to power we multiply the exponents"

Examples: Simplify.

Ex: $(-2a)^4 = (-2)^4 a^4 \leftarrow \text{Property \#4}$
 $= \boxed{16a^4}$

Ex: $(-2a)^5 = (-2)^5 a^5 = \boxed{-32a^5}$

Ex: $(3xy)^4 = 3^4 x^4 y^4 = \boxed{81x^4y^4}$

Ex: $(x^3y^4)^5 = (x^3)^5 (y^4)^5$
 $= \boxed{x^{15}y^{20}} \leftarrow \text{Property \#2}$

Ex: $\left(\frac{y^3}{x}\right)^2 = \frac{(y^3)^2}{(x)^2} = \boxed{\frac{y^6}{x^2}}$

Ex: $(4x^2)^5 = 4^5 x^{10} = \boxed{1024x^{10}}$

5.2.2

Ex: $(-3x^3y^4z)^2 = (-3)^2 x^6 y^8 z^2 = \boxed{9x^6 y^8 z^2}$

Ex: $\frac{(2x^2yz)(-x^4y^2z^3)^4}{(3xy)^2}$

$= \frac{2x^2yz(-1)^4 x^{16} y^8 z^{12}}{3^2 x^2 y^2}$

$= \frac{2(-1)^4 x^{18} y^9 z^{13}}{9x^2 y^2} = \frac{2(1) x^{16} y^7 z^{13}}{9}$

$= \boxed{\frac{2x^{16} y^7 z^{13}}{9}}$

OR $\boxed{\frac{2}{9} x^{16} y^7 z^{13}}$

5.3: Definitions of b^0 and b^{-n}

5.3.1

(Division with Exponents and Negative Exponents)

Definition of a negative exponent

$$x^{-n} = \frac{1}{x^n}$$

Note: $x^{-n} = \frac{x^{-n}}{1} = \frac{1}{x^n}$

Also: $\frac{1}{x^{-n}} = \frac{x^n}{1}$

$$x^{-1} = \frac{x^{-1}}{1} = \frac{1}{x^1} = \frac{1}{x} \text{ (so } x^{-1} \text{ is the reciprocal of } x \text{)}$$

Examples: Simplify. Write all answers without negative exponents.

Ex: $3^{-2} = \frac{3^{-2}}{1} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$

Ex: $-2^{-4} = -\frac{2^{-4}}{1} = -\frac{1}{2^4} = \boxed{-\frac{1}{16}}$

Ex: $(-3)^{-4} = \frac{(-3)^{-4}}{1} = \frac{1}{(-3)^4} = \boxed{\frac{1}{81}}$

5.3.2

Ex. $\frac{1}{4^{-2}} = \frac{4^2}{1} = \frac{16}{1} = \boxed{16}$

Ex. $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}}$

$$= \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$$

Ex. $2x^{-5}$

$$= \frac{2x^{-5}}{1}$$

$$= \boxed{\frac{2}{x^5}}$$

Ex. $(2x)^{-5}$

$$= \frac{(2x)^{-5}}{1}$$

$$= \frac{2^{-5}x^{-5}}{1}$$

$$= \frac{1}{2^5 x^5}$$

$$= \boxed{\frac{1}{32x^5}}$$

Ex. $\frac{ab^{-2}}{c^{-3}}$

$$= \boxed{\frac{ac^3}{b^2}}$$