5.3 Negative Exponents (cont'd)

From RR 9

a) $-3xy^{-4} = \dfrac{-3xy^{-4}}{1} = \boxed{\dfrac{-3x}{y^4}} = \boxed{-\dfrac{3x}{y^4}}$

b) $(-3xy)^{-4} = \dfrac{(-3xy)^{-4}}{1} = \dfrac{(-3)^{-4}x^{-4}y^{-4}}{1} = \dfrac{1}{(-3)^4x^4y^4}$

$= \boxed{\dfrac{1}{81x^4y^4}}$

Recall:

x^{-n} is defined to be $\dfrac{1}{x^n}$

c) $(-3x^2y^{-3})^{-4} = \dfrac{(-3x^2y^{-3})^{-4}}{1}$

$= \dfrac{(-3)^{-4}x^{-8}y^{12}}{1} = \dfrac{y^{12}}{(-3)^4x^8} = \boxed{\dfrac{y^{12}}{81\,x^8}}$

d) $\dfrac{1}{(-3x^2y^{-3})^{-4}} = \dfrac{1}{(-3)^{-4}x^{-8}y^{12}} = \dfrac{(-3)^4x^8}{y^{12}} = \boxed{\dfrac{81x^8}{y^{12}}}$

Important Fact:

$x^0 = 1$, as long as $x \neq 0$.

Here's why:

$$\frac{2^3}{2^3} = \frac{8}{8} = 1$$

must be equal

Recall:

$$\frac{x^a}{x^b} = x^{a-b}$$

From the properties of exponents,

$$\frac{2^3}{2^3} = 2^{3-3} = 2^0$$

Those two approaches need to give us the same answer. So,

$$2^0 = 1.$$

Also:

$$2^5 = 32$$

$$2^4 = 32 \div 2 = 16$$

$$2^3 = 16 \div 2 = 8$$

$$2^2 = 8 \div 2 = 4$$

$$2^1 = 4 \div 2 = 2$$

$$2^0 = 2 \div 2 = 1 \leftarrow \text{So } 2^0 = 1$$

$$2^{-1} = 1 \div 2 = \frac{1}{2}$$

Ex: $(-5)^0 = \boxed{1}$

5.3.5

Ex: $(4x^2y^3z)^0 = 1$

as long as $x \neq 0$, $y \neq 0$, $z \neq 0$

Note: 0^0 is undefined.

Ex: Simplify. Write all answers without negative exponents.

$$\left(\frac{2x}{3}\right)^{-4} = \frac{2^{-4}x^{-4}}{3^{-4}} = \frac{3^4}{2^4x^4} = \boxed{\frac{81}{16x^4}}$$

Ex:

$$\frac{4(3x^{-1}y^2z^{-3})^{-1}}{(5y^{-3})^2} = \frac{4\cdot 3^{-1}x^{1}y^{-2}z^{3}}{5^2y^{-6}}$$

$$= \frac{4\quad x\quad z^3\quad y^6}{3^1\quad y^2\quad \cdot 5^2}$$

$$= \frac{4xy^6z^3}{25\cdot 3y^2} = \boxed{\frac{4xy^4z^3}{75}}$$

$\boxed{5.3.6}$

Ex:

$$\frac{-3x^{-5}(-x^{-1}y^{3}z^{-2})^{-4}}{(3x^{2}y^{4})^{-2}(2xy^{-3}z)^{4}}$$

$$= \frac{-3x^{-5}(-1)^{-4}x^{4}y^{-12}z^{8}}{3^{-2}x^{-4}y^{-8}\cdot 2^{4}x^{4}y^{-12}z^{4}}$$

$$= \frac{-3 \quad x^{4} \quad z^{8}\cdot 3^{2}x^{4}y^{8} \quad y^{12}}{x^{5}(-1)^{4} \quad y^{12} \quad \cdot 2^{4}x^{4} \quad z^{4}}$$

$$= \frac{-3\cdot 9\, x^{4}x^{4}y^{8}y^{12}z^{8}}{(-1)^{4}\cdot 16\, x^{5}x^{4}y^{12}z^{4}} = \frac{-27x^{8}y^{20}z^{8}}{1\cdot 16x^{9}y^{12}z^{4}}$$

$$= \boxed{\frac{-27y^{8}z^{4}}{16x}} = \boxed{-\frac{27y^{8}z^{4}}{16x}}$$

5.5: Addition and Subtraction of Polynomials

A polynomial expression, or polynomial, is a sum of terms of the form ax^n, where a is a real number and n is a nonnegative integer.

Polynomials: $3x^3 + 4x^2 + 8$

$\frac{1}{2}x^4 - \frac{1}{3}x^7$

$x^6 - 2$

Not polynomials: $\frac{1}{x^2} + \frac{1}{x^5}$ ← (can be written $x^{-2} + x^{-5}$)

Polynomials must have positive integer exponents

$3\sqrt{x} + \sqrt[3]{x}$ ← $\left(3x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)$

The term with the largest exponent is called the leading term.

The coefficient of the leading term is the leading coefficient.

The degree is the largest exponent.

If the polynomial has two or more variables, the degree is the largest total exponent.

Ex: Consider $2x^3 - 7x^5 + 12x^4 - 18$

Degree: 5 This is a 5^{th} degree polynomial.

Leading term: $-7x^5$

Leading coefficient: -7

Often polynomials are written in order of descending powers:

$$-7x^5 + 12x^4 + 2x^3 - 18$$

Ex: What is the degree of

$$\underbrace{2x^2y^3}_{\substack{\text{total}\\\text{exponent}\\2+3=5}} - \underbrace{5x^3y^3}_{\substack{\text{total}\\\text{exp}\\3+3=6}} + \underbrace{4xy^6}_{\substack{\text{total}\\\text{exponent}\\1+6=7}} - \underbrace{8y^3}_{\substack{\text{total exponent}\\3}} ?$$

largest total exponent

$$\boxed{\text{The degree is 7.}}$$

Special terms for certain polynomials:

A polynomial with 3 terms is a trinomial.

A polynomial with 2 terms is a binomial

A polynomial with 1 term is a monomial.

Ex: $4x^2 - 8x + 1$ is a 2^{nd} degree trinomial.

$3 - 11x^4$ is a 4^{th} degree binomial

$2x^3y^5z$ is a 9^{th} degree monomial