5.6: Multiplying Polynomials

5.6.1

Note Title 2/28/2018

Example: $8x^2(9x^4)$ simplify.

$= \boxed{72x^6}$ (we just multiplied two monomials)

multiplying a monomial times a polynomial

Ex: $3x^3(2x^2 - 8x + 7)$ multiply.

$= 3x^3(2x^2) + 3x^3(-8x) + 3x^3(7)$

$= \boxed{6x^5 - 24x^4 + 21x^3}$

Ex: $-5x^2y^3(-xy^4 + 8xy - 3x^3y^5 - 2y^2)$

$= -5x^2y^3(-xy^4) - 5x^2y^3(8xy) - 5x^2y^3(-3x^3y^5) - 5x^2y^3(-2y^2)$

$= \boxed{5x^3y^7 - 40x^3y^4 + 15x^5y^8 + 10x^2y^5}$

multiplying two polynomials with two or more terms

Recall: $c(a+b) = ca + cb$

$(a+b)(c) = ac + bc$

Ex: $(\underbrace{3x}_{a} + \underbrace{5}_{b})\underbrace{(x-8)}_{c}$

$= \underbrace{3x}_{a}\underbrace{(x-8)}_{c} + \underbrace{5}_{b}\underbrace{(x-8)}_{c}$

$= 3x(x-8) + 5(x-8)$

$= 3x^2 - 24x + 5x - 40 = \boxed{3x^2 - 19x - 40}$

Ex: $(x^2+6)(4x-7)$

$= x^2(4x-7) + 6(4x-7)$

$= \boxed{4x^3 - 7x^2 + 24x - 42}$

Ex: $(-2x+8)(x^2+7x+4)$

$= -2x(x^2+7x+4) + 8(x^2+7x+4)$

$= -2x^3 - 14x^2 - 8x + 8x^2 + 56x + 32$

$= \boxed{-2x^3 - 6x^2 + 48x + 32}$

Ex: $(x^2-7x-1)(2x^2-5x-6)$

$= x^2(2x^2-5x-6) - 7x(2x^2-5x-6) - 1(2x^2-5x-6)$

$= 2x^4 - 5x^3 - 6x^2$
$\quad - 14x^3 + 35x^2 + 42x$
$\quad - 2x^2 + 5x + 6$

$= \boxed{2x^4 - 19x^3 + 27x^2 + 47x + 6}$

Special Products & Patterns

The "FOIL" method (for multiplying two binomials)

F: First

O: Outer

I: Inner

L: Last

Ex: $(2x+3)(3x-4)$

(F)irst (L)ast (I)nner (O)uter

$= 6x^2 - 8x + 9x - 12$

$= \boxed{6x^2 + x - 12}$

Difference of Two Squares Pattern

Ex: $(x-6)(x+6)$

$= x^2 + \cancel{6x} - \cancel{6x} - 36$

$= \boxed{x^2 - 36}$

Difference of Two Squares Pattern

$$(a+b)(a-b) = a^2 - b^2$$

Notice: $a^2 - a\cancel{b} + a\cancel{b} - b^2 \quad = a^2 - b^2$

Ex: $(x-7)(x+7) = \boxed{x^2 - 49}$

Ex: $(3x+5)(3x-5) = \boxed{9x^2 - 25}$

Note: Sometimes $3x+5$ and $3x-5$ are called conjugates.

Perfect Squares

Ex: $(x-8)^2$

$= (x-8)(x-8)$

$= x^2 - 8x - 8x + 64$

$= \boxed{x^2 - 16x + 64}$

5.4: Scientific Notation

We write numbers so they have exactly 1 digit to the left of the decimal point, multiplied by a power of 10.

Ex: Write 3254 in scientific notation.

$$3254 = 3.254 \times 10^3$$

3.254.

Ex: Write 0.001536 in scientific notation

$$0.001536 = 1.536 \times 10^{-3}$$

0.001536

3 slots

5.7: Division of Polynomials

Dividing a polynomial by a monomial

Note:

$$\frac{2}{7}+\frac{3}{7}=\frac{2+3}{7}$$

$$=\frac{5}{7}$$

Ex: Divide. $\dfrac{6x^4-8x^3+12x^2-9}{2x^2}$

Break into separate fractions and reduce.

$$\frac{6x^4-8x^3+12x^2-9}{2x^2}=\frac{\cancel{6}^{3}x^4}{\cancel{2}_{1}x^2}-\frac{\cancel{8}^{4}x^3}{\cancel{2}_{1}x^2}+\frac{\cancel{12}^{6}x^2}{\cancel{2}_{1}x^2}-\frac{9}{2x^2}$$

$$=\frac{3x^2}{1}-\frac{4x}{1}+\frac{6}{1}-\frac{9}{2x^2}$$

$$=\boxed{3x^2-4x+6-\frac{9}{2x^2}}$$

5.7.2

Polynomial Long Division

Use this when divisor has 2 or more terms.

Review: Divide $\frac{379}{12}$.

$$\begin{array}{r} 31 \\ 12\overline{)379} \\ \underline{-36} \\ 19 \\ \underline{-12} \\ 7 \end{array}$$

So $\frac{379}{12} = \boxed{31 + \frac{7}{12}}$

Terminology

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor}\overline{)\text{Dividend}} \\ \vdots \\ \overline{\text{Remainder}} \end{array}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$

In our example: $(31)(12) + 7$

$= 372 + 7 = 379 =$ dividend

$$\begin{array}{r} 31 \\ \times 12 \\ \hline 62 \\ 31 \\ \hline 372 \end{array}$$

Ex: Divide.

5.7.3

$$\frac{3x^2+7x+9}{x+2}$$

$\frac{3x^2}{x}$ $\qquad$ $\frac{1x}{x}$

$$
\begin{array}{r|l}
 & 3x \quad + 1 \\
\hline
x+2 & 3x^2 + 7x + 9 \\
 & -(3x^2 + 6x) \\
\hline
 & \qquad\quad 1x + 9 \\
 & \qquad -(1x + 2) \\
\hline
 & \qquad\qquad\quad 7
\end{array}
$$

$3x(x+2)=3x^2+6x$

$1(x+2) = x+2$

write answer

$$\frac{3x^2+7x+9}{x+2} = \boxed{3x+1+\frac{7}{x+2}}$$

Ex: Divide.

$$\frac{x^3-6x^2+x+14}{x-5}$$

$$\frac{x^3}{x} \qquad \frac{-x^2}{x} \qquad \frac{-4x}{x}$$

$$\begin{array}{r|llll}
 & x^2 & -x & -4 & \\
\hline
x-5 & x^3 & -6x^2 & +x & +14 \\
 & -(x^3 & -5x^2) & & \\
\hline
 & & -x^2 & +x & \\
 & & -(-x^2 & +5x) & \\
\hline
 & & & -4x & +14 \\
 & & & -(-4x & +20) \\
\hline
 & & & & -6
\end{array}$$

$x^2(x-5)$

$-x(x-5)$

$$\frac{x^3-6x^2+x+14}{x-5} = \boxed{x^2-x-4+\frac{-6}{x-5}}$$

$$= \boxed{x^2-x-4-\frac{6}{x-5}}$$

Check answer by multiplying:

$$(x-5)(x^2-x-4)-6$$

$$= x^3 - x^2 - 4x$$
$$\qquad -5x^2 + 5x + 20 \; -6$$

$$= x^3-6x^2+x+14 \checkmark$$