6.1: The Greatest Common Factor and Factoring by Grouping

Note Title 3/19/2018

Chapter 6: Factoring Polynomials

$(x+3)(x+4)$ ← Factored form

$= x^2 + 7x + 12$ ← unfactored form

Numbers: 14

$= 2 \cdot 7$ ← Factored form

To find the Greatest Common Factor (GCF)

* Find the largest number that divides into all the coefficients.
* Find highest power of each variable that is a factor of all the terms
  (if a variable appears in all the terms, we choose the smallest exponent)
* multiply the numerical part and the variable part to get the GCF.

Example: Factor out the GCF.

GCF: $3x^2$

$15x^4 - 6x^2$

$= 3x^2\left(\frac{15x^4}{3x^2} - \frac{6x^2}{3x^2}\right)$

$= \boxed{3x^2(5x^2 - 2)}$

Check by multiplying: $3x^2(5x^2 - 2)$

$= 15x^4 - 6x^2$ ✓ matches original

Ex: Factor.

$48x^3 - 32x^2$

GCF: $8x^2$

$= 8x^2(6x - 4)$ — can still factor out 2

Check: $8x^2(6x-4)$

$= 48x^3 - 32x^2$ ✓ matches the original

$= 8x^2(6x-4)$

$= 8x^2 \cdot 2(3x-2)$ (Factor out 2)

$= \boxed{16x^2(3x-2)}$

OR, get GCF $= 16x^2$ from the beginning:

GCF: $16x^2$

$48x^3 - 32x^2$

$= \boxed{16x^2(3x-2)}$

Ex: Factor.

$24x^3 + 18x^2 - 6$

GCF: 6

$= \boxed{6(4x^3 + 3x^2 - 1)}$

Check: $24x^3 + 18x^2 - 6$ ✓

Ex: Factor.

$-30x^4y^3z + 12x^3y^6 - 42x^4y^2z^2$

GCF: $6x^3y^2$

$= 6x^3y^2\left(\frac{-30x^4y^3z}{6x^3y^2} + \frac{12x^3y^6}{6x^3y^2} - \frac{42x^4y^2z^2}{6x^3y^2}\right)$

$= \boxed{6x^3y^2(-5xyz + 2y^4 - 7xz^2)}$

Check it by multiplying!

Factoring by Grouping

6.1.3

Example: Factor.

$3x^3 - 7x^2 + 15x - 35$ GCF: 1

$= (3x^3 - 7x^2) + (15x - 35)$

$= x^2(3x - 7) + 5(3x - 7)$ [Factor out the GCF from each set of parentheses]

$= \boxed{(3x-7)(x^2+5)}$ or $\boxed{(x^2+5)(3x-7)}$

Check: $(3x-7)(x^2+5)$

$= 3x^3 + 15x - 7x^2 - 35$ ✓ ok

Ex: Factor.

$4x^3 - 3x^2 - 36x + 27$ GCF: 1

$= (4x^3 - 3x^2) + (-36x + 27)$

$= x^2(4x - 3) - 9(4x - 3)$

$= (4x-3)(x^2 - 9)$

$= \boxed{(4x-3)(x+3)(x-3)}$

Check: $(4x-3)(x^2-9)$

$= 4x^3 - 36x - 3x^2 + 27$ ✓

Note: $(x+3)(x-3)$

$= x^2 - 3x + 3x - 9$

$= x^2 - 9$ ✓

Ex: Factor.

$$-16x^2y - 4x^2 + 24xy + 6x \qquad \text{GCF: } 2x$$

$$= 2x(-8xy - 2x + 12y + 3) \quad \text{(check by multiplying)}$$

$$= 2x[(-8xy - 2x) + (12y + 3)]$$

$$= 2x[-2x(4y + 1) + 3(4y + 1)]$$

$$= 2x[(4y+1)(-2x+3)]$$

$$= \boxed{2x(4y+1)(-2x+3)}$$

Another way:

Factor. $-16x^2y - 4x^2 + 24xy + 6x$

$$= (-16x^2y - 4x^2) + (24xy + 6x)$$

$$= -4x^2(4y + 1) + 6x(4y + 1)$$

$$= (4y+1)(-4x^2 + 6x)$$

can still factor out $2x$

$$= (4y+1)(2x)(-2x + 3)$$

$$= \boxed{2x(4y+1)(-2x+3)}$$

Also correct: $\boxed{-2x(4y+1)(2x-3)}$

6.2: Factoring Trinomials with leading coefficient 1

Ex: Factor.

$$x^2+8x+15$$

$$\boxed{(x+3)(x+5)}$$

GCF: 1

F: First

O: Outer

I: Inner

L: Last

Check:

$$(x+3)(x+5)$$
$$= x^2+5x+3x+15$$
$$= x^2+8x+15$$

Ex: $x^2+10x-24$ (− signs are opposite)

$$\boxed{(x+12)(x-2)}$$

Check:

$$x^2+12x-2x-24$$
$$= x^2+10x-24$$

If my signs are opposite, I want a difference of 10x

24
1 · 24
2 · 12
3 · 8
4 · 6

Ex: $x^2-10x+24$ (+ signs are the same)

$$\boxed{(x-4)(x-6)}$$

If signs are the same, I want a sum of 10x

24
1 · 24
2 · 12
3 · 8
4 · 6

Check:

$$x^2-6x-4x+24$$
$$x^2-10x+24$$