6.5: Difference of Two Squares and Perfect Square Trinomials.

Note Title 3/28/2018

6.4: Book covers the AC method (grouping method) for factoring trinomials.

For the 6.4 homework, work the problems using the method we already know.

6.5:

<u>Recall:</u> $(a+b)(a-b)$

$= a^2 - \cancel{ab} + \cancel{ab} - b^2$

$= a^2 - b^2$

☆☆☆ ☆☆☆

Difference of Two Squares Factorization:

$$\boxed{a^2 - b^2 = (a+b)(a-b)}$$

<u>Ex:</u> Factor. $x^2 - 25$

[use $a^2-b^2=(a+b)(a-b)$ with $a=x$ and $b=5$]

$= (x)^2 - (5)^2$

$= (x+5)(x-5)$

<u>Check:</u> $(x+5)(x-5)$

$= x^2 - 5x + 5x - 25$

$= x^2 - 25$ ✓

<u>Perfect Squares:</u>

$1^2=1$	$5^2=25$	$9^2=81$
$2^2=4$	$6^2=36$	$10^2=100$
$3^2=9$	$7^2=49$	$11^2=121$
$4^2=16$	$8^2=64$	$12^2=144$

Ex: $y^2 - 16$

$= (y)^2 - (4)^2$

$= \boxed{(y-4)(y+4)}$ OR $\boxed{(y+4)(y-4)}$

Check: $y^2 + \cancel{4y} - \cancel{4y} - 16$

$y^2 - 16$ ✓

Ex: Factor.

$4x^2 - 9$

$= (2x)^2 - (3)^2$

$= \boxed{(2x+3)(2x-3)}$

Check: $4x^2 - \cancel{6x} + \cancel{6x} - 9$

$= 4x^2 - 9$ ✓

Ex: $100y^2 - 81z^2$

$= (10y)^2 - (9z)^2$

$= \boxed{(10y+9z)(10y-9z)}$

or $\boxed{(10y-9z)(10y+9z)}$

Check it.

Ex: $x^2 - 64$

$\boxed{(x+8)(x-8)}$

Ex: $x^2 + 49$

Try: $(x+7)(x-7)$

Check: $x^2 - \cancel{7x} + \cancel{7x} - 49$

$= x^2 - 49$ No!

Try: $(x+7)(x+7)$

check: $x^2 + 7x + 7x + 49$

$= x^2 + 14x + 49$ No!

$\boxed{x^2 + 49 \text{ is prime.}}$ (It does not factor)

Why is $x^2 + 49$ prime? (6.5.3)

Write as a trinomial: $x^2 + 0x + 49$

(+) same signs
want a sum of $0x$

49
$1 \cdot 49$
$7 \cdot 7$

A sum of 0 is impossible.

Important Fact: $a^2 + b^2$ is always prime.
$x^2 +$ positive is always prime.

Ex: Factor.

$$32x^2 - 50y^2$$
$$= 2(16x^2 - 25y^2)$$
$$= 2((4x)^2 - (5y)^2)$$
$$= \boxed{2(4x+5y)(4x-5y)}$$

Check it..

Ex: Factor.

$$x^5 - x$$
$$= x(x^4 - 1)$$
$$= x(x^2+1)(x^2-1)$$
$$= \boxed{x(x^2+1)(x+1)(x-1)}$$

Ex: $x^4 - 13x^2 + 36$

$$= (x^2 - 9)(x^2 - 4)$$
$$= \boxed{(x+3)(x-3)(x+2)(x-2)}$$

Check!

Perfect Square Trinomials

Note: $(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

Check: $(a+b)(a+b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

(+) same signs
sum of 12x

Ex: $x^2 + 12x + 36$

$= (x+6)(x+6)$

$= \boxed{(x+6)^2}$

x^2 ← $x \cdot x$

36 ← $1 \cdot 36$, $2 \cdot 18$, $3 \cdot 12$, $4 \cdot 9$, $6 \cdot 6$

6x

6x

Ex: $(x+9)^2 = x^2 + 18x + 81$

$(x-5)^2 = x^2 - 10x + 25$

$(x-5)(x-5)$

$= x^2 - 5x - 5x + 25$

$= x^2 - 10x + 25$

Ex: Factor. $16x^2 - 56xy + 49y^2$

Try: $\boxed{(4x - 7y)^2}$ ← answer

Check: $(4x-7y)(4x-7y)$

$16x^2 - 28xy - 28xy + 49y^2$

$16x^2 - 56xy + 49y^2$ Yes!

$\boxed{6.5.5}$

Ex: $4x^2 - 20x + 9$

Try: $(2x - 3)^2$

check: $(2x-3)(2x-3)$

$4x^2 - 6x - 6x + 9$

$4x^2 - 12x + 9$ No!

$4x^2 - 20x + 9$

$$\boxed{(2x-1)(2x-9)}$$

Check: $4x^2 - 18x - 2x + 9$

$4x^2 - 20x + 9$ ✓

$4x^2$

$4x \cdot x$

$2x \cdot 2x$

9

$1 \cdot 9$

$3 \cdot 3$

$2x$

$18x$

6.7: Solving Quadratic Equations by Factoring

A quadratic equation (in x) is an equation that can be written as $ax^2+bx+c=0$, where a, b, c are real numbers and $a \neq 0$.

$$\underbrace{ax^2}_{\text{quadratic term}} + \underbrace{bx}_{\text{linear term}} + \underbrace{c}_{\text{constant term}} = 0$$

is called standard form for a quadratic equation

Zero Product Property (or Zero Product Theorem)

If the product of two numbers is 0, then at least one of the numbers must be 0.

In other words, if $AB=0$, then $A=0$ or $B=0$.

Ex:

Solve.

6.7.2

$$x^2 - 24 = 10x$$
$$\quad -10x \qquad -10x$$

(−) opp signs difference of 10

$$x^2 - 10x - 24 = 0$$

[write in std. form $ax^2 + bx + c = 0$]

$$(x + 2)(x - 12) = 0$$

[Apply Zero Product Property]

$x + 2 = 0$ or $x - 12 = 0$

$\quad -2 \quad -2 \qquad\qquad +12 \quad +12$

$x = -2 \qquad x = 12$

Solution Set: $\boxed{\{-2, 12\}}$

24
1·24
2·12
3·8
4·6

Check: $(x+2)(x-12)$
$x^2 - 12x + 2x - 24$
$x^2 - 10x - 24$ ✓

Ex: Solve.

$$2x^3 - 14x^2 + 24x = 0$$
$$2x(x^2 - 7x + 12) = 0$$
$$2x(x - 3)(x - 4) = 0$$

$2x = 0$ or $x - 3 = 0$ or $x - 4 = 0$

$\frac{2x}{2} = \frac{0}{2} \qquad x = 3 \qquad x = 4$

$x = 0$

Sol'n Set: $\boxed{\{0, 3, 4\}}$

Ex: Solve:

$$2x^2 = 18$$
$$-18 \quad -18$$
$$2x^2 - 18 = 0$$

$$2(x^2-9)=0$$

$$2(x+3)(x-3)=0$$

$2=0$ <u>or</u> $x+3=0$ or $x-3=0$

never true

$-3 \quad -3$ $\qquad$ $+3 \quad +3$

$x=-3$ $\qquad$ $x=3$

Sol'n Set: $\boxed{\{-3,3\}}$

or $\boxed{\{\pm 3\}}$