3.4: Slope-Intercept Form of a Linear Equation (3.4.1)

Note Title 2/5/2015

Standard Form of a Line: $Ax + By = C$, where A and B are not both 0.

Ex: $2x - 3y = -5$ is in standard form.

Slope-intercept Form of a Line:

$y = mx + b$, where m is the slope and $(0, b)$ (or just b) is the y-intercept.

Note: To find the y-intercept, set $x = 0$ and solve for y:

$x = 0$

$y = mx + b$

$y = m(0) + b$

$y = 0 + b$

$y = b \Rightarrow (0, b)$ ordered pair.

Example: Write the equation of the line that has slope $-\frac{3}{2}$ and y-intercept 4. Graph it.

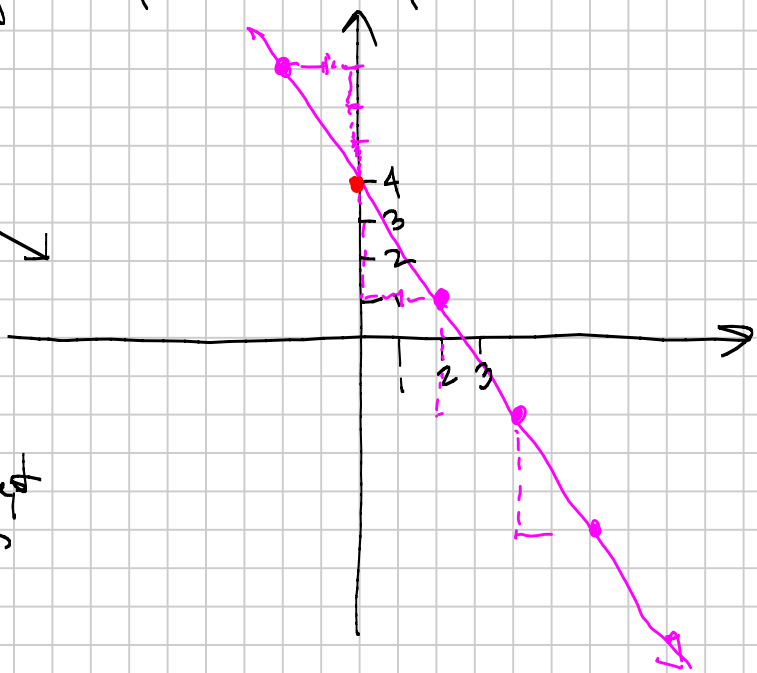
y-intercept 4 $\Rightarrow$ ordered pair $(0, 4)$

$y = mx + b$ (slope-intercept form)

m = slope $= -\frac{3}{2}$

b = y-intercept $\Rightarrow (0, 4)$ so $b = 4$

$$y = -\frac{3}{2}x + 4$$ eqn in slop-intercept form



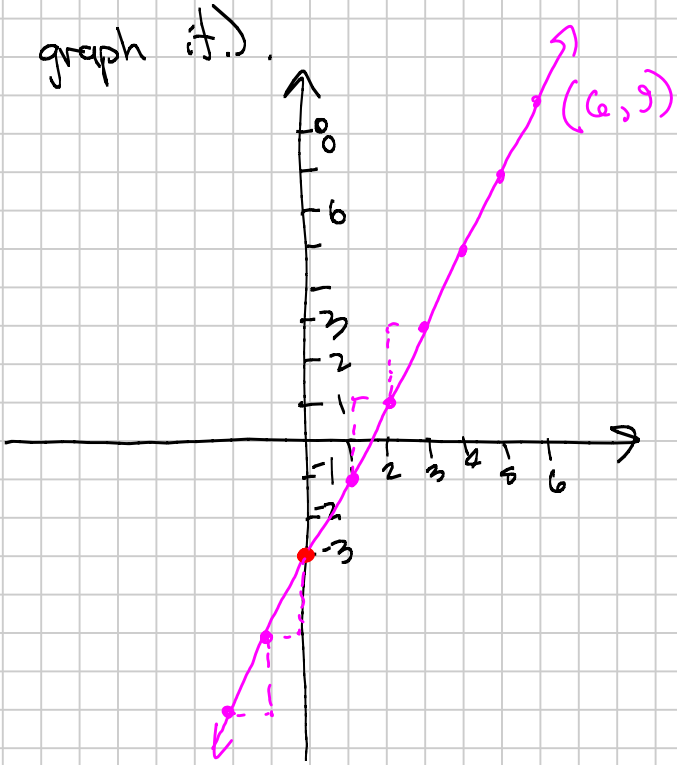
Ex: Graph the equation $y = 2x - 3$.

(Let's use slope-intercept form to graph it.)

$y = mx + b$ It's already in slope-intercept form.

slope: $m = 2 = +\frac{2}{1}$ ↗ +

y-intercept: $b = -3 \Rightarrow$ ordered pair $(0, -3)$



Let's do a spot-check:

Is $(6, 9)$ a solution?

$y = 2x - 3$

$x = 6, y = 9$ $\quad 9 = 2(6) - 3$

$9 = 12 - 3$

$9 = 9$ ✓ok

Example: Graph the line $2x + 3y = 9$ by writing in slope-intercept form.

Want to get $y = mx + b$.

Need to solve for y: $\quad 2x + 3y = 9$

$-2x \qquad -2x$

$3y = -2x + 9$

$\frac{3y}{3} = \frac{-2x}{3} + \frac{9}{3}$

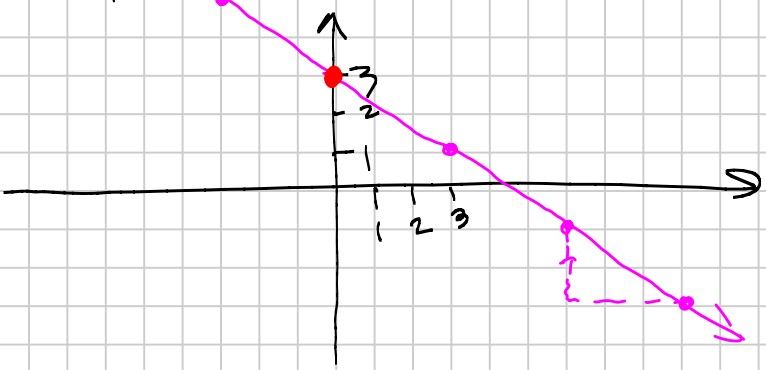
$\boxed{y = -\frac{2}{3}x + 3}$ slope-intercept form

$y = mx + b$

$y = -\frac{2}{3}x + 3$

Slope: $m = -\frac{2}{3}$ ↘ −

y-intercept: $3 \Rightarrow (0, 3)$

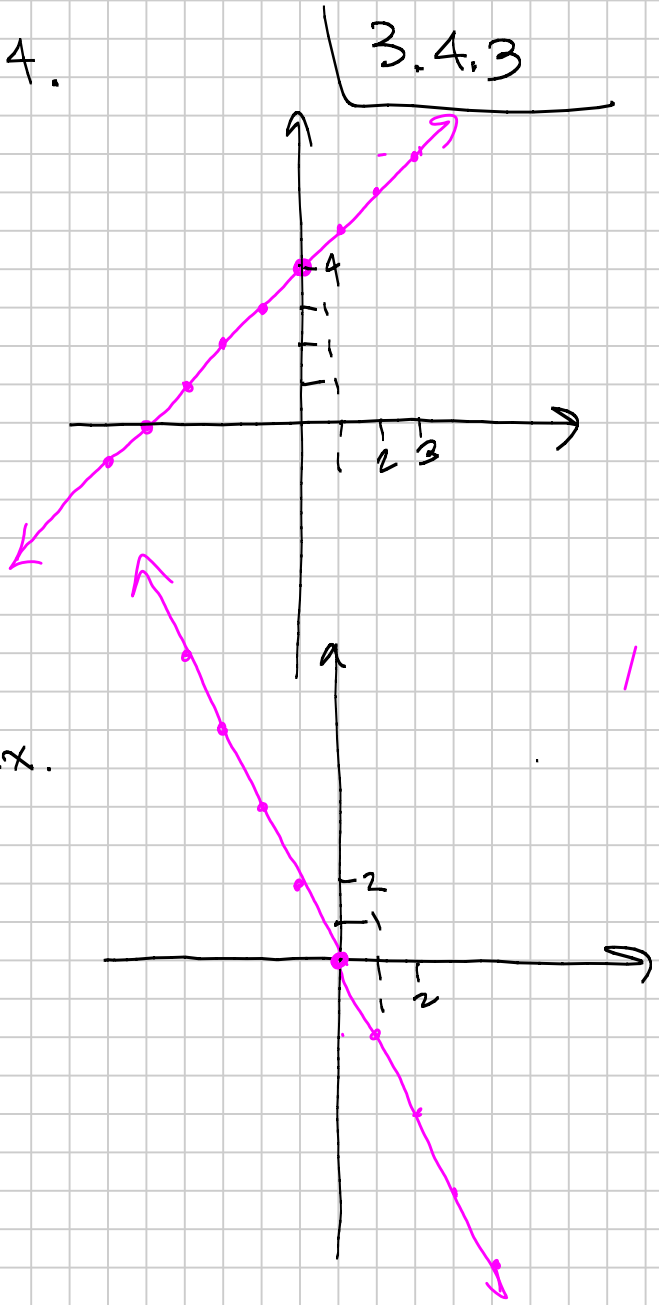


Ex: Graph the line $x - y = -4$.

$$x - y = -4$$
$$-x \quad -x$$
$$-y = -x - 4$$
$$\frac{-1y}{-1} = \frac{-x}{-1} - \frac{4}{-1}$$
$$y = x + 4$$
$$y = 1x + 4$$

slope: $m = +1 = +\frac{1}{1}$ ↗

y-intercept: $b = 4 \Rightarrow (0, 4)$



Ex: Graph the line $y = -2x$.

$$y = mx + b$$
$$y = -2x + 0$$

slope: $m = -2 = -\frac{2}{1}$ ↘

y-intercept: $b = 0 \Rightarrow (0, 0)$

Parallel and Perpendicular Lines

* Parallel lines have the <u>same slope</u>.
* Perpendicular lines have slopes that are <u>opposite reciprocals</u>.

(Perpendicular lines intersect at a 90° angle (right angle, square angle)

Ex: Are these lines parallel, perpendicular, or neither?

a) $y = 2x + 5$
$y = 2x - 7$

$m_1 = 2$
$m_2 = 2$
} same slope, so **parallel**

b) $y = -\frac{3}{4}x + 4$
$y = \frac{4}{3}x$

$m_1 = -\frac{3}{4}$
$m_2 = \frac{4}{3}$
} **perpendicular**

c) $y = 3x - 7$
$y = \frac{1}{3}x + 4$

$m_1 = 3$
$m_2 = \frac{1}{3}$
} **neither**

4.1: Solving Linear Systems using graphing

System of equations: Group of equations

Solution to a system: A set of values for the variables that makes **all** the equations true.

We will work with systems of 2 linear equations in 2 variables.

Ex: Is $(9,-2)$ a solution of the system?

$$\begin{cases} 2x + 5y = 8 \\ 3x - 2y = 23 \end{cases}$$

Put $x=9$, $y=-2$ into $2x+5y=8$:

$$2(9)+5(-2)=8$$
$$18-10=8$$
$$8=8 \quad \checkmark \text{ True}$$

It works in 1st eqn

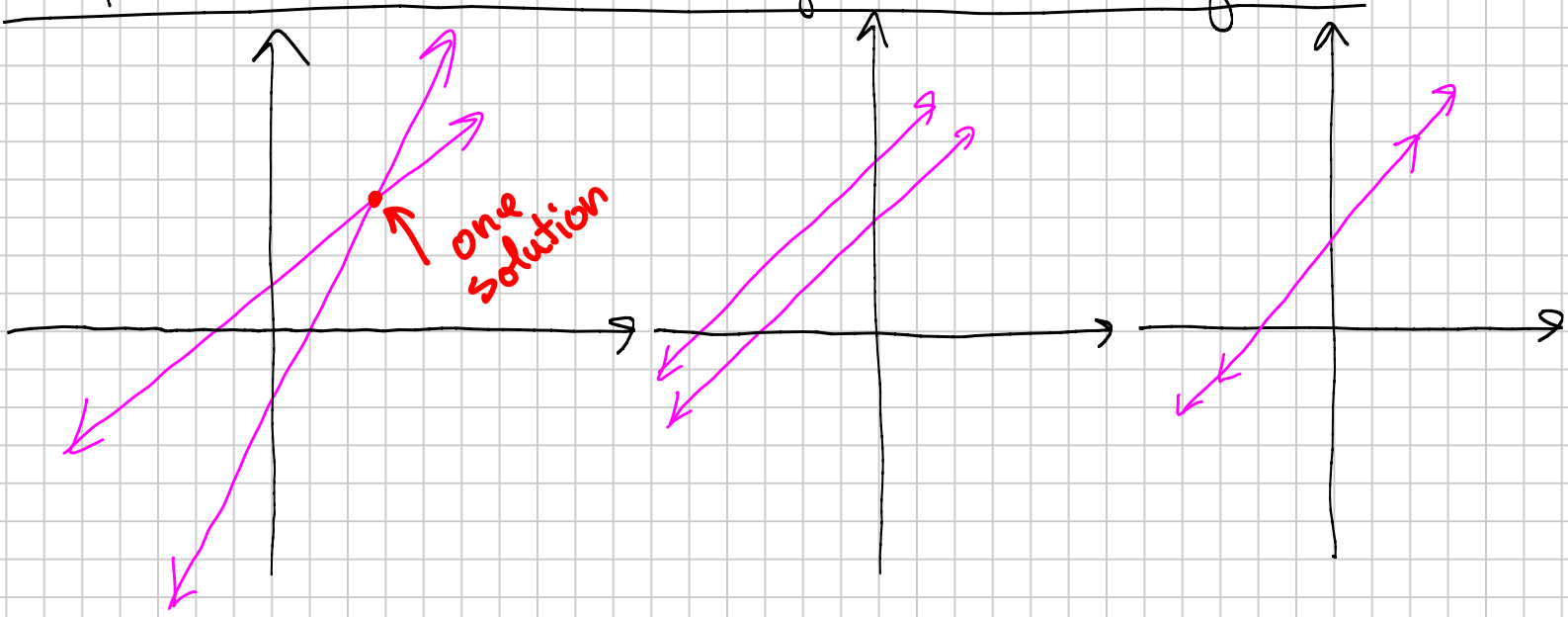
Put $x=9$, $y=-2$ into $3x-2y=23$:

$$3(9)-2(-2)=23$$
$$27+4=23$$
$$31=23 \quad \text{false.}$$

No, $(9,-2)$ is not a solution to the system.

4.1.2

3 possible situations for a system of 2 linear equations



Independent System

one solution

(lines intersect at a single point)

Inconsistent System

no solution

(lines are parallel)

Dependent System

infinitely many solutions

(both equations represent the same line)

To solve a system by graphing, graph both lines and then estimate the solution from the graph.

Question on Test #4

For an inconsistent system, or an independent system, or a dependent system:

1) State the number of solutions.
2) Describe the graph in words (lines are parallel, lines intersect at one point, lines are the same)
3) Illustrate with an example graph.