4.2: Solving Linear Systems with the Substitution Method

Note Title 4/23/2018

Solving by Substitution: Step-by-step

1) Solve one equation (your choice) for one variable (your choice).
2) Substitute this expression into the other equation.
3) Solve for the remaining variable.
4) Plug this value into either equation and solve.
5) Check your solution.

4.2.2

Example: Solve the system by substitution.

$$\begin{cases} x - y = 9 \\ 2x - 11y = -18 \end{cases}$$

1) Solve $x - y = 9$ for x:

$+y \quad +y$

$$x = y + 9$$

2) Substitute $x = y + 9$ into $2x - 11y = -18$.

$$2(\quad) - 11y = -18$$

$$2(y + 9) - 11y = -18$$

3) Solve for y:

$$2y + 18 - 11y = -18$$

$$-9y + 18 = -18$$

$-18 \quad -18$

$$-9y = -36$$

$$\frac{-9y}{-9} = \frac{-36}{-9}$$

$$y = 4$$

4) Plug $y = 4$ into $x - y = 9$.

$$x - 4 = 9$$

$+4 \quad +4$

$$x = 13$$

Solution: $(13, 4)$

This is an independent system.

5) Check your solution.

$x - y = 9$

$x = 13, y = 4 \Rightarrow 13 - 4 = 9$

$9 = 9$ ✓ True!

$2x - 11y = -18$

$x = 13, y = 4 \Rightarrow 2(13) - 11(4) = -18$

$26 - 44 = -18$

$-18 = -18$ ✓ True!

4.2.3

Ex: Solve by substitution.

$$\begin{cases} -4x + 2y = -4 \\ 2x + y = 8 \end{cases}$$

1) Solve $2x + y = 8$ for y.

$-2x \quad -2x$

$$y = -2x + 8$$

2) Put $y = -2x + 8$ into $-4x + 2y = -4$

$$-4x + 2(\quad) = -4$$

$$-4x + 2(-2x + 8) = -4$$

3) Solve for x:

$$-4x - 4x + 16 = -4$$

$$-8x + 16 = -4$$

$-16 \quad -16$

$$-8x = -20$$

$$\frac{-8x}{-8} = \frac{-20}{-8}$$

$$x = \frac{20}{8} = \frac{5}{2}$$

4) Plug $x = \frac{5}{2}$ into $-4x + 2y = -4$

$$-4\left(\frac{5}{2}\right) + 2y = -4$$

$$-\frac{20}{2} + 2y = -4$$

$$-10 + 2y = -4$$

$+10 \quad +10$

$$2y = 6$$

$$\frac{2y}{2} = \frac{6}{2}$$

$$y = 3$$

Solution: $\left(\frac{5}{2}, 3\right)$

Previous example cont'd:

5) Check your answer!

$$-4x + 2y = -4$$

$x = \frac{5}{2}, y = 3$

$$-4\left(\frac{5}{2}\right) + 2(3) = -4$$

$$-10 + 6 = -4$$

$$-4 = -4 \quad \checkmark$$

$$2x + y = 8$$

$x = \frac{5}{2}, y = 3$

$$2\left(\frac{5}{2}\right) + 3 = 8$$

$$5 + 3 = 8$$

$$8 = 8 \quad \checkmark \text{ True!}$$

4.3: Solving Linear Systems by the Addition Method (Elimination Method)

Steps for solving by elimination/addition

1) Choose a variable to eliminate. Then multiply one or both equations by a strategic number, so the variable will disappear when you add the equations.
   (look for the least common multiple of the coefficients)

2) Add the equations. One variable should disappear.

3) Solve for the remaining variable.

4) Either (a) Repeat Steps 1-3 for the other variable
   or (b) Plug your value into either equation and solve.

5) Check your solution.

Ex: Solve the system using the elimination (addition) method.

$$\begin{cases} 11x - 5y = 2 \\ 3x - y = 1 \end{cases}$$

$11x - 5y = 2 \longrightarrow 11x - 5y = 2$

$3x - y = 1 \xrightarrow{(-5)} -15x + 5y = -5$

Add: $-4x + 0 = -3$

$-4x = -3$

$\frac{-4x}{-4} = \frac{-3}{-4}$

$x = \frac{3}{4}$

Could plug $x = \frac{3}{4}$ into either eqn, or do elimination again to get 2nd variable:

$11x - 5y = 2 \xrightarrow{(3)} 33x - 15y = 6$

$3x - y = 1 \xrightarrow{(-11)} -33x + 11y = -11$

Add: $0 - 4y = -5$

$-4y = -5$

$\frac{-4y}{-4} = \frac{-5}{-4}$

$y = \frac{5}{4}$

Solution: $\left(\frac{3}{4}, \frac{5}{4}\right)$

Previous example cont'd:

Check your solution:

$x = \frac{3}{4}$, $y = \frac{5}{4}$

$11x - 5y = 2$

$11\left(\frac{3}{4}\right) - 5\left(\frac{5}{4}\right) = 2$

$\frac{33}{4} - \frac{25}{4} = 2$

$\frac{8}{4} = 2$

$2 = 2$ ✓ OK!

$x = \frac{3}{4}$, $y = \frac{5}{4}$

$3x - y = 1$

$3\left(\frac{3}{4}\right) - \frac{5}{4} = 1$

$\frac{9}{4} - \frac{5}{4} = 1$

$\frac{4}{4} = 1$

$1 = 1$ ✓

Ex: Solve the system.

$$\begin{cases} -x + 6y = 2 \\ 3x - 18y = 5 \end{cases}$$

$-x + 6y = 2 \xrightarrow{(3)} -3x + 18y = 6$

$3x - 18y = 5 \longrightarrow 3x - 18y = 5$

Add $0x + 0y = 11$

$0 = 11$ False!

No Solution

This is an inconsistent system (lines are parallel)

Ex: Solve the system

$$2x - 10y = 6$$
$$25y - 5x = -15$$

$2x - 10y = 6 \xrightarrow{(5)} 10x - 50y = 30$

$-5x + 25y = -15 \xrightarrow{(2)} -10x + 50y = -30$

Add $0x + 0y = 0$

$0 = 0$ Always true!

Infinitely many solutions

Dependent system (lines are the same)