5.1: Discrete Random Variables and Probability Distributions

A *random variable* is a quantitative variable that represents the outcomes of a probability experiment. Thus, the value of a random variable depends on chance.

A *discrete random variable* is a random variable that takes on a finite or countably infinite number of values.

A *continuous random variable* is a random variable that takes on all values on an interval of the real number line (i.e., it is not countable).

A *discrete probability distribution* is a function that assigns a probability to each outcome. (So, it assigns a probability to each value of the discrete random variable). If there are a finite number of outcomes, the sum of all their probabilities must equal 1. Each probability must be between 0 and 1, inclusive. The probability distribution can be described by a table, graph, or mathematical formula.

Notation:

If X is a random variable, then the probability of X taking on the value x is denoted $P(X = x)$. For example, the probability of X taking on the value 3 is $P(X = 3)$. The probability of X taking on a values of at least 5 is denoted $P(X \geq 5)$.

Example 1: A probability distribution is given by the table below.

x	12	13	14	15	16	17	18
$P(X = x)$	0.32	0.18	0.13	0.11	0.10	0.08	0.08

sum = 1.0

a) What is $P(x = 17)$?

$P(x = 17) = \boxed{0.08}$

b) What is $P(x \geq 16)$?

$P(x \geq 16) = 0.10 + 0.08 + 0.08 = \boxed{0.26}$

c) What is $P(x > 13)$?

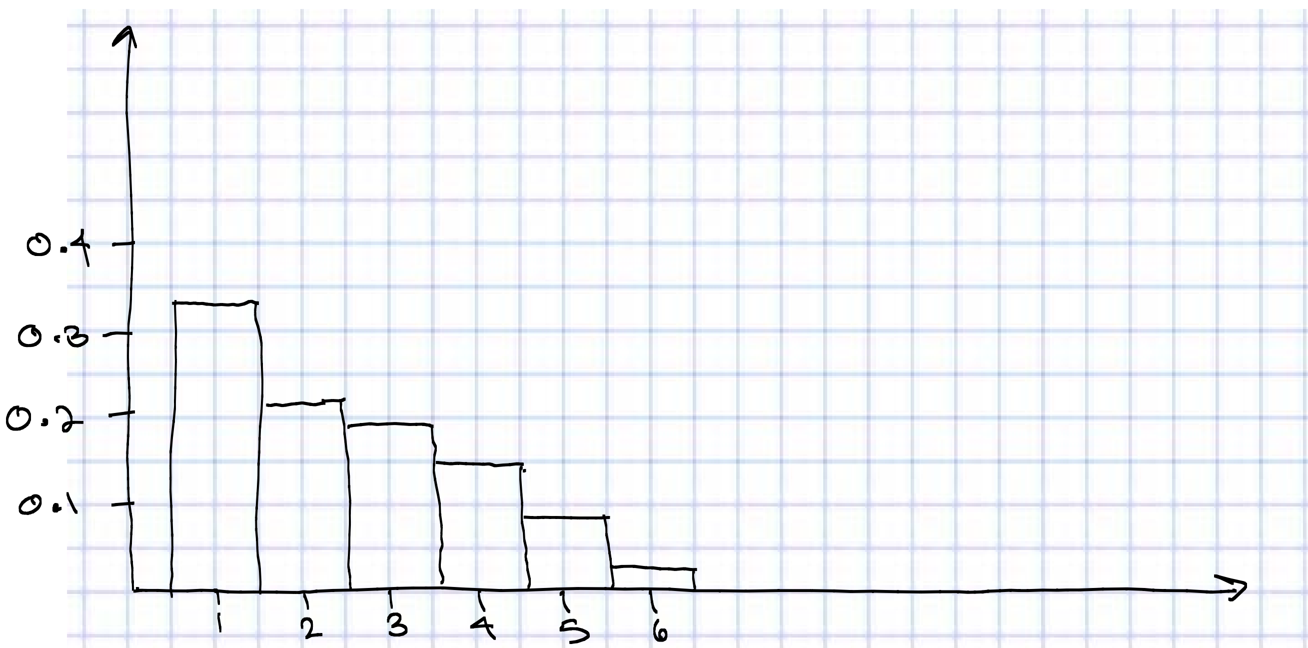
$P(x > 13) = 0.13 + 0.11 + 0.10 + 0.08 + 0.08$

$= \boxed{0.50}$

Example 2: A car repair shop's records show that 25 clients have 6 cars, 83 clients have 5 cars, 140 clients have 4 cars, 183 clients have 3 cars, and 209 clients have 2 cars. The remaining 313 clients own only 1 car. Determine the probability distribution for the number of cars owned by the shop's clients. Construct a probability histogram. If the manager decides to randomly call a customer and invite him or her to complete a satisfaction survey, what is the probability that the customer called has 2 or fewer cars?

X = number of cars owned by the client.

x	6	5	4	3	2	1	Total
Frequency	25	83	140	183	209	313	953 = n
$P(X=x)$	$\frac{25}{953} \approx 0.026$	$\frac{83}{953} \approx 0.087$	$\frac{140}{953} \approx 0.147$	0.192	0.219	0.328	0.999



b = boy
g = girl

Example 3: Create a probability distribution to represent the number of girls in a three-child family. Assume that boys and girls are equally likely. Construct the probability histogram. What is the probability that a three-child family has exactly one girl? What is the probability that a three-child family has at least one girl?

Sample space = $S = \{ggg, bgg, gbg, ggb, gbb, bgb, bbg, bbb\}$

Let X = number of girls.

x	3	2	1	0
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

2 girls: $\{bgg, gbg, ggb\}$