5.2: The Mean and Standard Deviation of a Discrete Random Variable

Mean of a discrete random variable:

The Mean (Expected value) of a Discrete Random Variable:

Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of X is

$$\mu = x_1 p_1 + x_2 p_2 + ... + x_n p_n .$$

Suppose an experiment is repeated many times, and the values of X are recorded and then averaged. As the number of repetitions increases, the average value of X will become closer and closer to μ. For that reason, the mean is called the *expected value* of X.

Example 1: A probability distribution is given by the table below. Find the mean (the expected value of X).

x	3	4	5	6	7	8	9
$P(X = x)$	0.15	0.20	0.30	0.12	0.08	0.10	0.05

Sum = 1.0

Expected value (mean): $\mu = E(X) = 3(0.15) + 4(0.20) + 5(0.30) + 6(0.12) + 7(0.08) + 8(0.10) + 9(0.05)$

$= \boxed{5.28}$

Example 2: Suppose that an organization sells 1000 raffle tickets for \$1 each. One ticket is for a gift basket worth \$200, and three tickets are for \$50 gift certificates to a restaurant. Find the expected net winnings for a person who buys one ticket.

X = net winnings for 1 ticket

Descriptions of outcome	X	$P(X=x)$
\$200 gift basket	$200-1=199$	$\frac{1}{1000} = 0.001 = P(X=199)$
\$50 gift cert	$50-1=49$	$\frac{3}{1000} = 0.003 = P(X=49)$
you lose	-1	$\frac{996}{1000} = 0.996 = P(X=-1)$

$1000-1-3=996$

$E(X) = \mu = 199(0.001) + 49(0.003) - 1(0.996) = \boxed{-\$0.65}$

<u>Example 3:</u> Suppose the yearly premium for a car insurance policy is \$2300 for a customer in a certain category. Statisticians for the insurance company have determined that a person in this category has a 0.007 probability of having an accident that costs the insurance company \$100,000 and a 0.015 probability of having an accident that costs the insurance company \$30,000. What is the expected value of the insurance policy to the customer? To the insurance company?

Standard deviation of a discrete random variable:

<u>The Standard Deviation of a Discrete Random Variable</u>**:**

Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the ~~mean~~ standard deviation of X is

$$\sigma = \sqrt{(x_1-\mu)^2 p_1 + (x_2-\mu)^2 p_2 + ... + (x_n-\mu)^2 p_n}$$

$$= \sqrt{\sum_{i=1}^{n}(x_i-\mu)^2 p_i}$$

So variance of X is $\sigma^2 = \sum_{i=1}^{n} (x_i-\mu)^2 p_i$

Example 4: Calculate the mean and standard deviation of the probability distribution.

x	$P(X = x)$ [sometimes written $P(x)$]	xp
0	0.11	0
1	0.32	0.32
2	0.43	0.86
3	0.10	0.3
4	0.04	0.16
		$\mu = 1.64$

Mean (expected value): $\mu = E(x) = 0(0.11) + 1(0.32) + 2(0.43) + 3(0.10) + 4(0.04)$

$= \boxed{1.64 = \mu}$

x	$x - \mu$	$(x-\mu)^2$	p	$(x-\mu)^2 p$
0	$0 - 1.64$	$(-1.64)^2 = 2.6896$	0.11	$2.6896(0.11) = 0.295856$
1	$1 - 1.64 = -0.64$	$(-0.64)^2 = 0.4096$	0.32	$0.4096(0.32) = 0.131072$
2	$2 - 1.64 = 0.36$	$(0.36)^2 = 0.1296$	0.43	0.055728
3	$3 - 1.64 = 1.36$	$(1.36)^2 = 1.8496$	0.10	0.18496
4	$4 - 1.64 = 2.36$	$(2.36)^2 = 5.5696$	0.04	0.222784

Sum: $\sigma^2 = 0.8904$

Variance:

$\sigma^2 = 0.8904$

Std. Dev.

$\sigma = \sqrt{0.8904}$

$= \boxed{0.9436}$

Example 5: Use the frequencies to construct a probability distribution for the random variable *X*, which represents the number of games bowled by customers at a bowling alley. Calculate the mean and standard deviation of *X*.

Number of Games	Frequency
1	37
2	45
3	29
4	12
5	4