Supplement: Basic Set Theory

Definition: A *set* is a well-defined collection of objects. Each object in a set is called an *element* of that set.

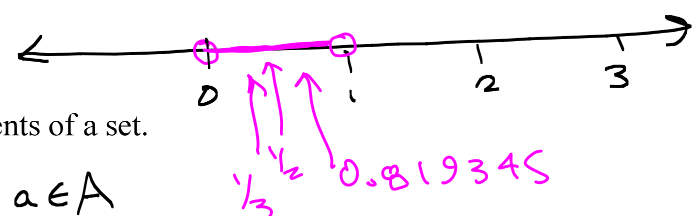
Examples of sets:
set of all positive integers (1, 2, 3, 4, 5, ...)
set of students enrolled in this class.
set of NBA players (currently on NBA teams)

Not sets:
all tall people
all cute dogs

Sets can be finite or infinite. Finite: has a fixed number of elements

Examples of finite sets:
set of NBA players
set of integers less than 100 (1, 2, 3, 4, ..., 99)

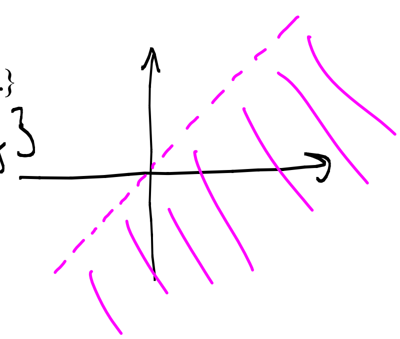
Examples of infinite sets:
set of all integers (..., -3, -2, -1, 0, 1, 2, 3, ...)
set of numbers between 0 and 1



Notation:

- We usually use capital letters for sets.
  We usually use lower-case letters for elements of a set.
- $a \in A$ means a is an element of the set A. $a \in A$
  $a \notin A$ means a is not an element of the set A. $a \notin A$
- The *empty set* is the set with no elements. It is denoted $\varnothing$. This is sometimes called the *null set*. $\varnothing$
- $S = \{x \mid P(x)\}$ means "S is the set of all x such that $P(x)$ is true". (called rule notation or set roster notation). "such that"

  Example: $S = \{x \mid x \text{ is an even positive integer}\}$ means $S = \{2, 4, 6, 8, \ldots\}$
- $n(A)$ means the number of elements in set A. $S = \{(x, y) \mid x > y\}$



Definition: We say two sets are *equal* if they have exactly the same elements.

Subsets:

Definition: If each element of a set *A* is also an element of set *B*, we say that *A* is a *subset* of *B*. This is denoted $A \subseteq B$ or $A \subset B$. If *A* is not a subset of *B*, we write $A \not\subset B$.

$A \subseteq B$ $A \not\subseteq B$

Definition: We say *A* is a *proper subset* of *B* if $A \subseteq B$ but $A \neq B$. (In other words, every element of *A* is also an element of *B*, but *B* contains at least one element that is not in *A*.)

Note on notation: Some books use the symbol $\subset$ to indicate a proper subset. Some books use $\subset$ to indicate any subset, proper or not.

Definition: The set of all elements under consideration is called the *universal set*, usually denoted *U*.
Example: If you're dealing with sets of real numbers, then *U* is the set of all real numbers. So "Wednesday" would not be an element of *U*, but 5.7 would be in *U*.

Example 1: Consider these sets.

$A = \{1,2,3,4,5,6\}$ $A \subseteq B$

$B = \{1,2,3,4,5,6,7,8\}$ $C \subseteq B$

$C = \{1,3,5,2,4,6\}$ $A = C$

Note:

- $\varnothing$ is a subset of every set. (i.e. $\varnothing \subseteq A$ for every set *A*.)
- Every set is a subset of itself. (i.e. $A \subseteq A$ for every set *A*.)

Example 2: List all subsets of $\{1,2,3\}$.

$\{1,2\}, \{1,3\}, \{2,3\}, \{1\}, \{2\}, \{3\}, \varnothing, \{1,2,3\}$

Note: If a set has *n* elements, how many subsets does it have? 2^n

So set $\{1,2,3\}$ has $2^3 = 8$ subsets

Set operations:

- Union $\cup$: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$ Key word: OR

  $A \cup B$
- Intersection $\cap$: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$ Key word: AND

  $A \cap B$
- Complement A' or A^C or $A^{\sim}$: $A' = \{x \in U \mid x \notin A\}$. Key word NOT

  A^c or A'

Note: $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.
$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.

Definition: We say that A and B are *disjoint sets* if $A \cap B = \varnothing$.

Example 3: $U = \{1,2,3,4,5,6,7,8\}$
$H = \{1,3,5,7\}$
$K = \{1,2,3\}$
$J = \{2,4,6,8\}$
$L = \{1,2\}$

$H \cap K = \{1,3\}$

$J \cup K = \{2,4,6,8,1,3\} = \{1,2,3,4,6,8\}$

$J \cap L = \{2\}$

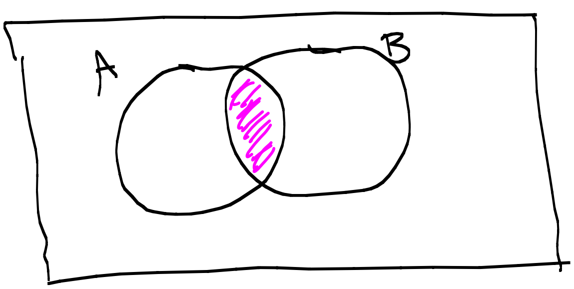
$J \cup L = \{1,2,4,6,8\}$

$H \cap J = \varnothing$

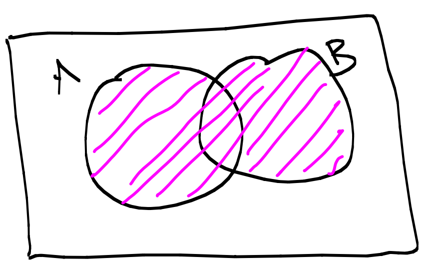
$L^c = \{3,4,5,6,7,8\}$

Venn Diagrams: These help us visualize set relationships and operations.

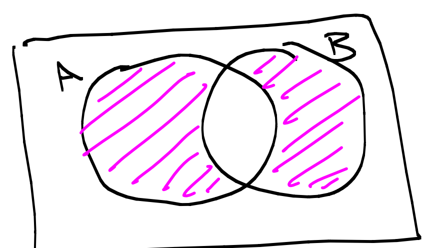
Example 4: Draw Venn diagrams for $A \cup B$, $A \cap B$, A^C, B^C, $(A \cap B)^C$, and $(A \cup B)^C$.



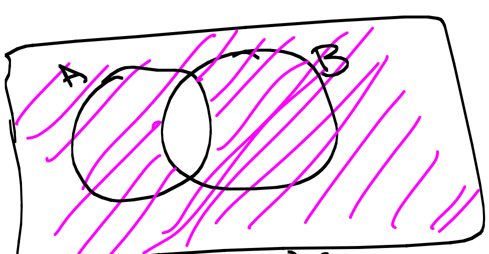
$A \cap B$



$A \cup B$



$(A \cap B^c) \cup (B \cap A^c)$



$(A \cap B)^c$