1.6: Properties of Real Numbers and Simplifying Expressions

1.6.1

Review - Properties of Real Numbers

Commutative Property of Addition: $a+b=b+a$

(Commutative means we can change the order)

Ex: $2+3=3+2$
(both equal 5)

Commutative Property of multiplication: $ab=ba$

Note: $ab = a \cdot b = a(b) = (a)(b)$

Ex: $2 \cdot 3 = 3 \cdot 2$
(both equal 6)

Note: Division and subtraction are not commutative operations
(cannot change the order)

Ex: $12-5 \neq 5-12$

$12-5=7$
$5-12=-7$

Ex: $\frac{8}{2} \neq \frac{2}{8}$

$\frac{8}{2}=4$

$\frac{2}{8}=\frac{1}{4}$

Associative Property of Addition: $a+(b+c)=(a+b)+c$

(associative means you can regroup)

Ex: $2+(3+4)=(2+3)+4$
$2+7 = 5+4$
$9 = 9$

Associative Property of Multiplication: $a(bc) = (ab)c$

Ex: $2(3 \cdot 4) = (2 \cdot 3) \cdot 4$

$2(12) = 6(4)$

$24 = 24$

Distributive Property of Real Numbers: $a(b+c) = ab + ac$

(multiplication distributes over addition)

$(a+b) \cdot c = ac + bc$

Ex. $2(3+4)$

$= 2(3) + 2(4)$

$= 6 + 8$

$= \boxed{14}$

Ex: $-3(4-12)$

$= -3(4) - 3(-12)$

$= -12 + 36$

$= \boxed{24}$

Algebraic Expression: Numbers and/or variables combined with arithmetic operations

Note: An expression does not have an equals sign.

Examples of expressions:

$2x+5$

$3x^2 + 7x - 2$

$4x^2y$

Algebraic expressions being added are called <u>terms</u>.
Algebraic expressions being multiplied are called <u>factors</u>.
The <u>coefficient</u> is the numerical factor in a term.

<u>Ex</u>: $4x^2 - 8x + 5$ has 3 terms.

The coefficient of the x^2 term is 4.

The coefficient of the x term is -8.

The constant (constant coefficient) is 5.

Combining Like Terms

When simplifying, we <u>combine like terms</u> by adding the coefficients.

<u>Like terms</u> have the same variables and exponents.

<u>Ex</u>: $2x^2, -5x^2, \frac{1}{9}x^2, 7x^2$ are <u>like terms</u>

<u>Ex</u>: $2x^2, 5x^3$ are <u>not</u> like terms.

<u>Ex</u>: Simplify.

$$2x^2 - 8x + 3 - 4x^3 + 7x^2 - 3x - \overbrace{x}^{-1x} + 1$$

$$= \boxed{-4x^3 + 9x^2 - 12x + 4}$$

Ex: $\frac{1}{3}x^2 - \frac{5}{2}x + 4 - 8x^2 - \frac{3}{4}x - \frac{11}{2}$

$= \frac{1}{3}x^2 - \frac{8}{1}x^2 - \frac{5}{2}x - \frac{3}{4}x + \frac{4}{1} - \frac{11}{2}$

$= \frac{1}{3}x^2 - \frac{8\cdot 3}{1\cdot 3}x^2 - \frac{5\cdot 2}{2\cdot 2}x - \frac{3}{4}x + \frac{4\cdot 2}{1\cdot 2} - \frac{11}{2}$

$= \frac{1}{3}x^2 - \frac{24}{3}x^2 - \frac{10}{4}x - \frac{3}{4}x + \frac{8}{2} - \frac{11}{2}$

$= \boxed{-\frac{23}{3}x^2 - \frac{13}{4}x - \frac{3}{2}}$

Ex: Simplify.

$2(m+5) - 4(3m^3 - 2m + 5m - 8)$

$= 2(m+5) - 4(3m^3 + 3m - 8)$

$= 2m + 10 - 12m^3 - 12m + 32$

$= \boxed{-12m^3 - 10m + 42}$ also correct: $-10m - 12m^3 + 42$

$42 - 10m - 12m^3$

Ex: $4 - 5a^2 + 2a - (-a^2 - 6a + 4)$

$= 4 - 5a^2 + 2a - 1(-1a^2 - 6a + 4)$

$= 4 - 5a^2 + 2a + a^2 + 6a - 4$

$= -4a^2 + 8a + 0$

$= \boxed{-4a^2 + 8a}$

Ex: Evaluate $x - y^2$ for $x=0, y=5$.

$$0 - 5^2$$

$$= 0 - 25$$

$$= \boxed{-25}$$

Ex: Evaluate $b^2 - 4ac$ for $a=7, b=-2, c=3$.

$$(-2)^2 - 4(7)(3)$$

$$= 4 - 28(3)$$

$$= 4 - 84$$

$$= \boxed{-80}$$

$$\begin{array}{r} {}^{2}28 \\ 3 \\ \hline 84 \end{array}$$

Ex: Simplify.

$$\frac{2}{3}\left(\frac{7}{2}x^2 - 4x - \frac{1}{4}\right)$$

$$= \frac{2}{3}\left(\frac{7}{2}x^2\right) + \frac{2}{3}\left(\frac{-4}{1}x\right) + \frac{2}{3}\left(-\frac{1}{4}\right)$$

$$= \frac{\cancel{14}^{7 \div 2}}{\cancel{6}_{\div 2\, 3}}x^2 - \frac{8}{3}x - \frac{\cancel{2}^{1}}{\cancel{12}_{6}}$$

$$= \boxed{\frac{7}{3}x^2 - \frac{8}{3}x - \frac{1}{6}}$$