

2.1: Addition, Subtraction, Multiplication,

2.1.1

Note Title

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and Division Properties of Equality

An equation is a statement that two values are equal.

It has an equals sign.

A solution to an equation is a value that makes the equation true.

To solve an equation means to find the solutions.

Ex. Is 3 a solution of $2y - 5 = 11$?

Substitute $y=3$: $2(3) - 5 = 11$

$$6 - 5 = 11$$

$$1 = 11 \text{ False!}$$

No, 3 is not a solution.

Is 8 a solution?

Substitute $y=8$: $2(8) - 5 = 11$

$$16 - 5 = 11$$

$$11 = 11 \text{ True!}$$

Yes, 8 is a solution.

Addition (and subtraction) Property of Equality

2.1.2

If $a = b$, then $a + c = b + c$.

This means we can add (or subtract) the same quantity on both sides.

Example: Solve.

$$x - 7 = 15$$

+7 +7

$$x + 0 = 22$$

$$x = 22$$

Solution Set:

$$\boxed{\{22\}}$$

Ex: Solve.

$$\frac{5}{8} = a + \frac{1}{8}$$

$-\frac{1}{8} \quad -\frac{1}{8}$

$$\frac{4}{8} = a$$

$$a = \frac{1}{2}$$

Sol'n Set: $\left\{\frac{1}{2}\right\}$

Multiplication (and Division) Property of Equality

$\neq$: not equal to

If $a=b$, then $ac=bc$ for $c \neq 0$.

This means we can multiply (or divide) both sides by the same nonzero number.

So, we also have: If $a=b$, then $\frac{a}{c} = \frac{b}{c}$ for $c \neq 0$.

Ex: Solve.

$$9x = 45$$

$$\frac{9x}{9} = \frac{45}{9}$$

$$x = 5$$

Sol'n set: $\boxed{\{5\}}$

Ex: Solve.

$$-8x = 24$$

$$\frac{-8x}{-8} = \frac{24}{-8}$$

$$x = -3$$

Sol'n set: $\boxed{\{-3\}}$

Ex: $-\frac{2}{3}y = 7$ Solve.

$$\frac{-\frac{2}{3}y}{-\frac{2}{3}} = \frac{7}{-\frac{2}{3}}$$

$$-\frac{3}{2} \left(-\frac{2}{3}y \right) = \frac{7}{1} \left(-\frac{3}{2} \right)$$

$$+ \frac{6}{6}y = -\frac{21}{2}$$

$$1y = -\frac{21}{2} \quad \text{or} \quad y = -10\frac{1}{2}$$

Recall: Dividing by a number is the same as multiplying by its reciprocal.

Sol'n set:

$$\left\{ -\frac{21}{2} \right\} \text{ or } \left\{ -10\frac{1}{2} \right\}$$

Ex: Solve.

$$-\frac{4}{3}x = -\frac{5}{7}$$

$$\frac{-\frac{4}{3}x}{-\frac{4}{3}} = \frac{-\frac{5}{7}}{-\frac{4}{3}}$$

$$x = -\frac{5}{7} \left(-\frac{3}{4}\right)$$

$$x = +\frac{15}{28}$$

Soln Set:

$$\boxed{\left\{\frac{15}{28}\right\}}$$

Ex:

$-\frac{2}{3}y = \frac{4}{5}$ Solve.

$$\frac{-\frac{2}{3}y}{-\frac{2}{3}} = \frac{\frac{4}{5}}{-\frac{2}{3}}$$

$$y = \frac{4}{5} \left(-\frac{3}{2}\right) = -\frac{12 \div 2}{10 \div 2} = -\frac{6}{5}$$

Sol'n:
Set

$$\boxed{\left\{-\frac{6}{5}\right\}}$$

Ex:

Solve. $0.35y = 8.25$

$$\frac{0.35y}{0.35} = \frac{8.25}{0.35}$$

$$y = \frac{8.25}{0.35} \left(\frac{100}{100}\right)$$

$$y = \frac{825 \div 5}{35 \div 5}$$

$$y = \frac{165}{7}$$

Soln Set: $\boxed{\left\{\frac{165}{7}\right\}}$

Ex: Solve.

$$\frac{0.35 + y}{-0.35} = \frac{8.25}{-0.35}$$

$$y = 7.90$$

$$\boxed{\{7.90\}}$$

Ex:

Solve

$$\frac{x}{7} = 6$$

2.1.5

$$\left(\frac{7}{1}\right)\left(\frac{x}{7}\right) = 6\left(\frac{7}{1}\right)$$

$$x = 42$$

or:

$$\frac{x}{7} = 6$$

rewrite:

$$\frac{1}{7}x = 6$$

$$\frac{\frac{1}{7}x}{\frac{1}{7}} = \frac{6}{\frac{1}{7}}$$

$$7\left(\frac{1}{7}x\right) = 6\left(\frac{7}{1}\right)$$

$$x = \frac{42}{1} = 42$$

Note: $\frac{x}{7} = \frac{1}{7}x$

why: $\frac{1}{7}x$

$$= \frac{1}{7} \cdot \frac{x}{1}$$

$$= \frac{x}{7}$$

2.2.1

Equivalent equations have the same solution set.

When solving an equation that requires several steps, each line should be equivalent to the original equation (and to all the other steps).

Solve.

$$-5x \qquad -5x$$

$$-2x + 7 = -9$$

$$\frac{-2x}{-2} = \frac{-16}{-2}$$

$$x = 8$$

$$\text{or } -2x = -9 - 7$$

$$-22 = -16$$

Sol'n Set: $\{\emptyset\}$

$$x=8 \Rightarrow 3(8)+7 = 5(8)-9$$

$$24 + 7 = 40 - 9$$

$$31 = 31$$

✓ ok!

2.2.2

Ex: Solve.

$$\begin{aligned}
 -5 - 2(y-5) &= 4y - 2(y+4) - 3 \\
 -5 - 2y + 10 &= 4y - 2y - 8 - 3 \\
 -2y + 5 &= 2y - 11 \\
 -2y & -2y \\
 -4y + 5 &= -11 \\
 -5 & -5 \\
 -4y &= -16 \\
 -4y &= -16 \\
 \underline{-4} & \quad \underline{-4} \\
 y &= 4
 \end{aligned}$$

Sol'n Set: {4}

Ex: Solve. $5x + 1 = 1$

$$5x = 0$$

$$\frac{5x}{5} = \frac{0}{5}$$

$$x = 0$$

Sol'n Set

{0}

Check it:

$$5x + 1 = 1$$

$$\begin{aligned}
 x=0 \Rightarrow 5(0) + 1 &= 1 \\
 0 + 1 &= 1 \\
 1 &= 1 \quad \checkmark
 \end{aligned}$$

$$\frac{0}{5} = 0 \quad \text{because } 5(?) = 0$$

Example: Solve.

$$4(7x - 2) = 2(14x - 1)$$

$$\begin{array}{r} 28x - 8 = 28x - 2 \\ -28x \quad -28x \end{array}$$

$$\begin{array}{r} -8 = -2 \\ +8 \quad +8 \end{array}$$

$$0 = 6$$

False! There is

No Solution

Ex.: Solve.

$$-2(3 - x) + 5x = 7x - 6$$

$$-6 + 2x + 5x = 7x - 6$$

$$-6 + 7x = 7x - 6$$

$$\begin{array}{r} -7x \quad -7x \end{array}$$

$$-6 = -6$$

$$\begin{array}{r} +6 \quad +6 \end{array}$$

$$0 = 0$$

True

Solution set is
all real numbers