

From 226

$$a) \frac{1}{2} \left(3x - \frac{1}{4} \right) + \frac{1}{4} = -\frac{7}{8}$$

$$\frac{1}{2} (\underline{\frac{3}{2}x}) + \frac{1}{2} \left(-\frac{1}{4} \right) + \frac{1}{4} = -\frac{7}{8}$$

$$\frac{3}{2}x - \frac{1}{8} + \frac{1}{4} = -\frac{7}{8}$$

$$\underline{8} \left(\frac{3}{2}x \right) - \underline{8} \left(\frac{1}{8} \right) + \underline{8} \left(\frac{1}{4} \right) = \underline{8} \left(-\frac{7}{8} \right)$$

$$12x - 1 + 2 = -7$$

$$12x + 1 = -7$$

$$12x = -7 - 1$$

$$12x = -8$$

$$\frac{12x}{12} = \frac{-8}{12}$$

$$x = -\frac{2}{3}$$

OR

$$8 \left(\frac{1}{2} \right) \left(3x - \frac{1}{4} \right) + 8 \left(\frac{1}{4} \right) = 8 \left(-\frac{7}{8} \right)$$

$$4 \left(3x - \frac{1}{4} \right) + 2 = -7$$

$$12x - 4 \left(\frac{1}{4} \right) + 2 = -7$$

$$12x - 1 + 2 = -7$$

same

$$\left\{ -\frac{2}{3} \right\}$$

2.8: Linear Inequalities

2.8.1

$<$ means "is less than"

$>$ means "is greater than"

$\leq$ means "is less than or equal to"

$\geq$ means "is greater than or equal to"

Note: $\leq$ and $\leq$ are equivalent.
 $\geq$ and $\geq$ are equivalent.

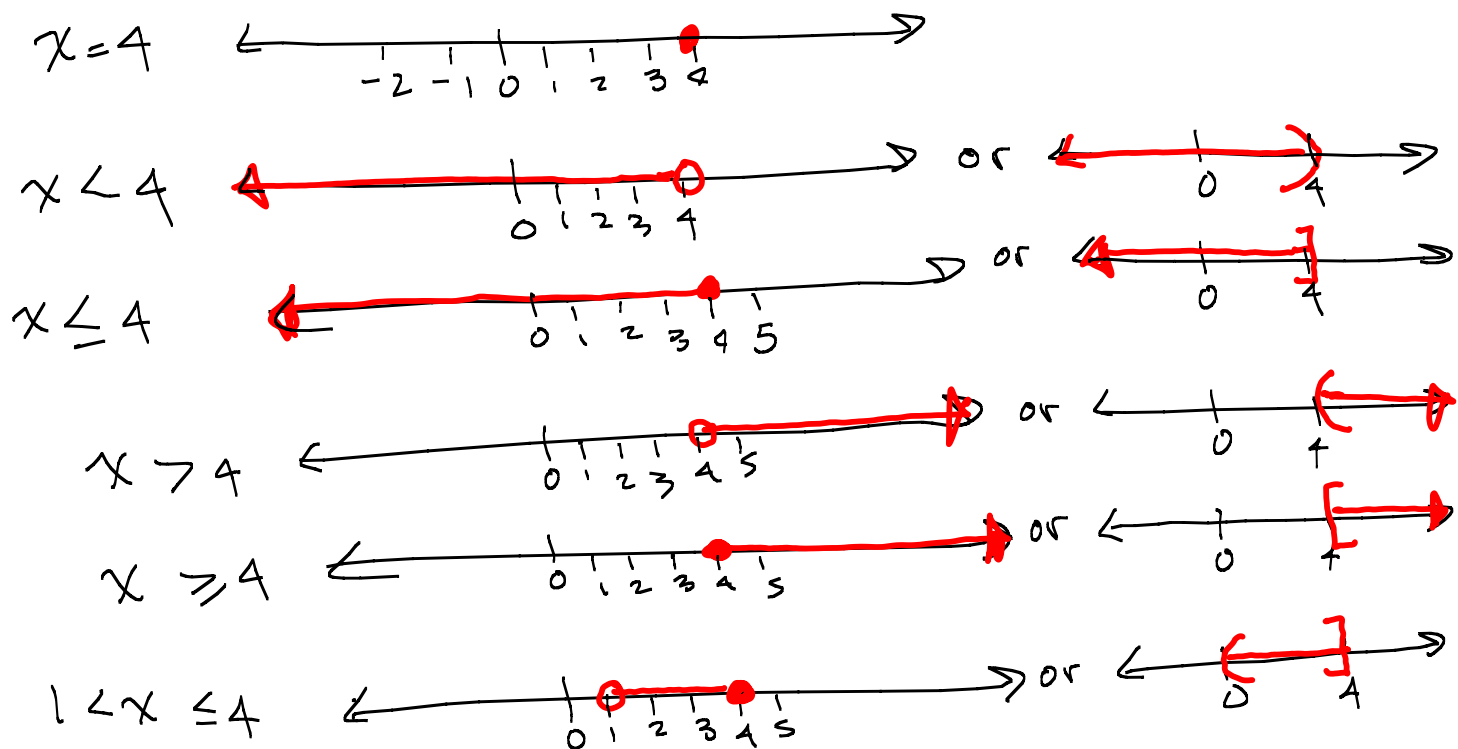
Definition: A linear inequality (in one variable) is any relationship that can be written in the

Form $ax + b < c$, $ax + b > c$
 $ax + b \leq c$, $ax + b \geq c$

where a, b, c are numbers and x is the variable.

Examples: $2x - 3 < 5$, or $6 \geq 2(6x + 1)$

We can use a number line to graph 2.8.2
the solution set of an inequality.



$1 < x \leq 4$ is a combined inequality.

$1 < x \leq 4$ means $1 < x$ and $x \leq 4$

Rewrite: $1 < x$ is
equivalent to $x > 1$.

Set-builder notation (set roster notation)

Inequality: $x \leq 4$

set builder notation: $\{x \mid x \leq 4\}$

↑
"such that"

The set of all numbers x such that $x \leq 4$,

Interval notation: We use a comma to separate the left and right boundaries of the shaded area.

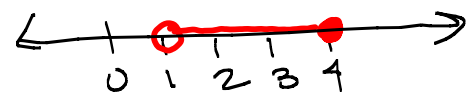
We use ∞ (infinity) for the far right "end" of the number line.

We use $-\infty$ for the left "end" of the number line.

Example: $1 < x \leq 4$

Set-builder notation: $\{x \mid 1 < x \leq 4\}$

Interval notation: $(1, 4]$



Set-builder

Graph

Interval notation

$$\{x \mid 1 < x < 4\}$$



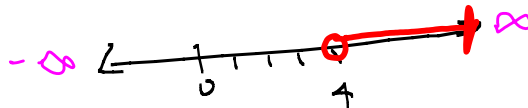
$$(1, 4)$$

$$\{x \mid x < 4\}$$



$$(-\infty, 4)$$

$$\{x \mid x > 4\}$$



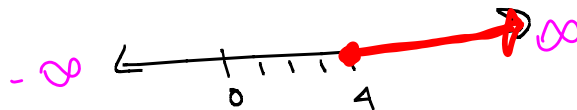
$$(4, \infty)$$

$$\{x \mid x \leq 4\}$$



$$(-\infty, 4]$$

$$\{x \mid x \geq 4\}$$



$$[4, \infty)$$

strict inequalities

Solving Linear Inequalities:

2.8.4

Addition Property of Inequality

If $a < b$, then $a + c < b + c$.

This means we can add (or subtract) the same quantity on both sides.

Ex. Solve.

$$x - 7 < 12$$

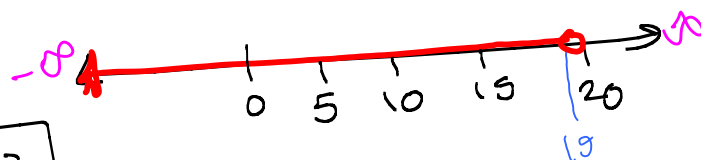
$$+7 \quad +7$$

$$x < 19$$

Set-builder: $\{x \mid x < 19\}$

Interval notation:

$$(-\infty, 19)$$



Ex. Solve.

$$3 < x + 4$$

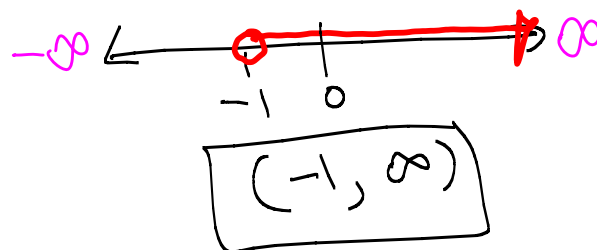
$$-4 \quad -4$$

$$-1 < x$$

$$x > -1$$

* * Rewrite with the variable on the left!

Set-builder: $\{x \mid x > -1\}$



Multiplicative Property of Equality

2.8.5

If $a < b$ and $c > 0$, then $ac < bc$.

If $a < b$ and $c < 0$, then $ac > bc$.

This means * we can multiply (or divide) both sides by the same positive number.

* we can multiply (or divide) both sides by the same negative number if we reverse the inequality sign.

Why?

$1 < 2$ true!

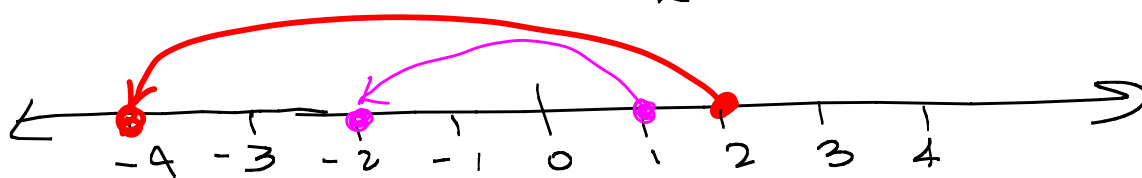
multiply by 2: $2(1) < 2(2)$

$2 < 4$ still true!

Start again with $1 < 2$ (still true)

multiply by -2: $-2(1) < -2(2)$

$-2 < -4$ False!



Ex:

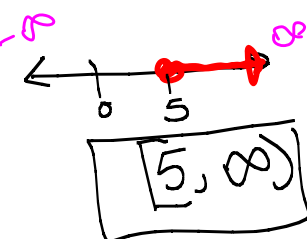
Solve

$$4x + 1 \geq 21$$

$$4x \geq 20$$

$$\frac{4x}{4} \geq \frac{20}{4}$$

$$x \geq 5$$



Ex: Solve.

$$-3x - 2 > 16$$

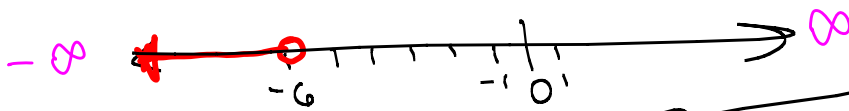
$$+2 \quad | +2$$

$$-3x > 18$$

Reverse the
inequality! $\rightarrow$

$$\frac{-3x}{-3} < \frac{18}{-3}$$

$$x < -6$$



$$(-\infty, -6)$$

$$\{ x \mid x < -6 \}$$