

2.8: Linear Inequalities (cont'd)

2.8.1

Note Title

9/19/2018

Example: Solve.

$$4 \leq 2(x - 12)$$

$$4 \leq 2x - 24$$

+24

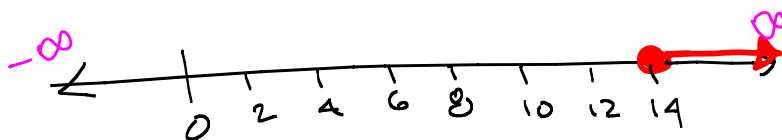
+24

$$28 \leq 2x$$

$$\frac{28}{2} \leq \frac{2x}{2}$$

$$14 \leq x$$

Rewrite: $x \geq 14$



Interval notation:

$$[14, \infty)$$

Set-builder notation:

$$\{x \mid x \geq 14\}$$

Ex. Solve.

$$4 - 5x > 3x + 1$$

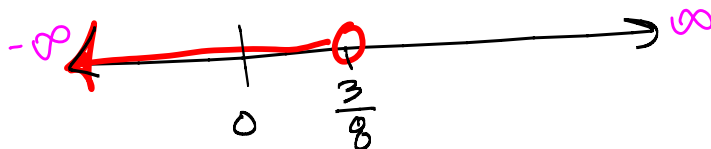
$$4 - 8x > 1$$

$$-8x > -3$$

$$\frac{-8x}{-8} < \frac{-3}{-8}$$

[Reverse the inequality sign!]

$$x < \frac{3}{8}$$



$$(-\infty, \frac{3}{8})$$

Ex.: Solve

2.8.8

$$-\frac{3}{4}x > 2$$

$$\frac{-\frac{3}{4}x}{-\frac{3}{4}} < \frac{2}{-\frac{3}{4}}$$

$$x < 2 \left(-\frac{4}{3}\right)$$

$$x < -\frac{8}{3}$$

Rewrite as mixed number:

$$-\frac{8}{3} = -2\frac{2}{3}$$

$$\boxed{(-\infty, -\frac{8}{3})} \text{ or } \boxed{(-\infty, -2\frac{2}{3})}$$

$$\text{or } -\frac{3}{4}x > 2$$

$$(4)\left(-\frac{3}{4}x\right) > 2(4)$$

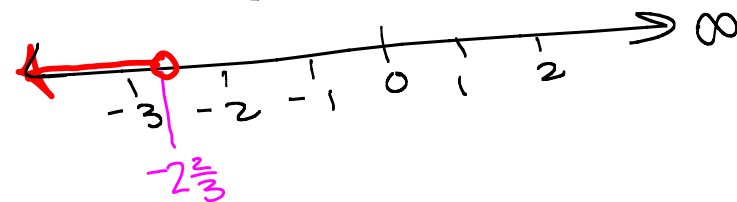
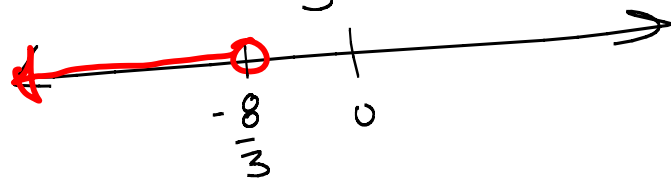
[multiply both sides by 4]

$$-3x > 8$$

$$\frac{-3x}{-3} < \frac{8}{-3}$$

(switch the inequality sign)

$$x < -\frac{8}{3}$$



$$\text{Ex: } 4(1-x) \leq 2(-2x+3) - 2 + x$$

$$4 - 4x \leq -4x + 6 - 2 + x$$

$$4 - 4x \leq -3x + 4$$

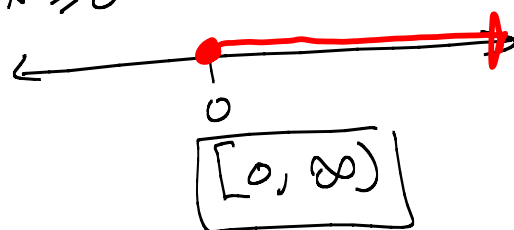
$$+4x \quad +4x$$

$$4 \leq x + 4$$

$$-4 \quad -4$$

$$0 \leq x$$

Rewrite: $x \geq 0$



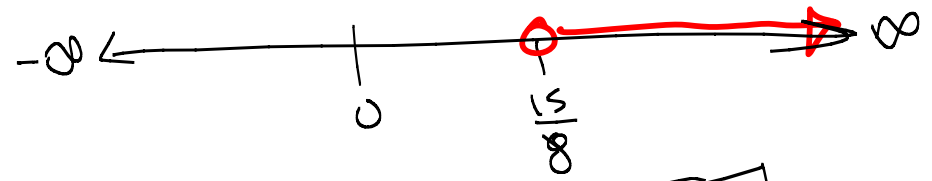
Ex. $-\frac{5}{2} > -\frac{4}{3}a$

$$\frac{-\frac{5}{2}}{-\frac{4}{3}} < \frac{-\frac{4}{3}a}{-\frac{4}{3}}$$

$$-\frac{5}{2} \left(-\frac{3}{4} \right) < a$$

$$\frac{15}{8} < a$$

Rewrite: $a > \frac{15}{8}$



$\left(\frac{15}{8}, \infty \right)$

Combined Inequalities

Example: Solve. $-7 \leq 2x+5 < 3$

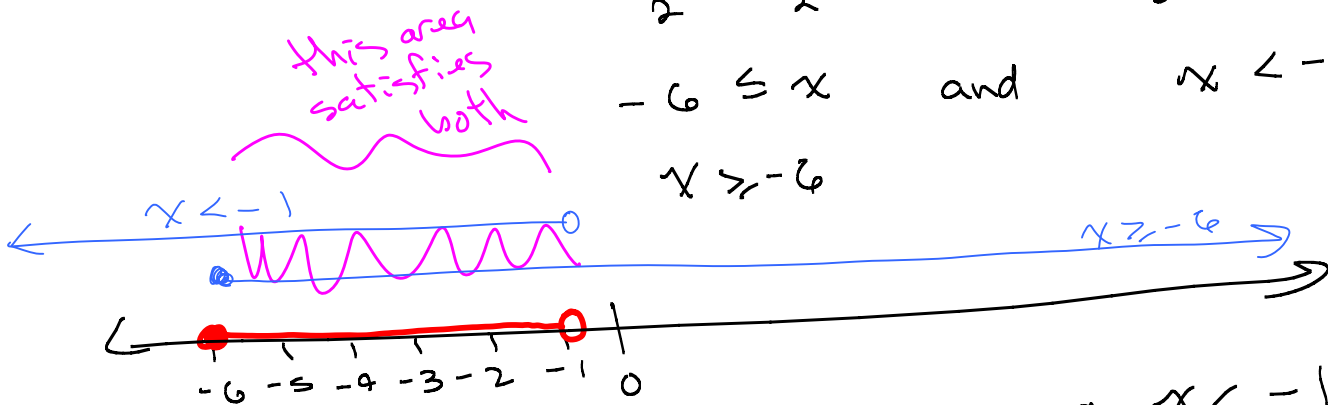
This means: $-7 \leq 2x+5$ and $2x+5 < 3$
 -5 -5 -5 -5

$$-12 \leq 2x \quad \text{and} \quad 2x < -2$$

$$\frac{-12}{2} \leq \frac{2x}{2} \quad \frac{2x}{2} < \frac{-2}{2}$$

$$-6 \leq x \quad \text{and} \quad x < -1$$

$$x \geq -6$$



Interval notation:

$$[-6, -1)$$

$-6 \leq x$ and $x < -1$
 Put these back together:

$$-6 \leq x < -1$$

Set-builder:

$$\{x \mid -6 \leq x < -1\}$$

OR

Keep them together:

(subtract 5 from all 3 sides)

$$-7 \leq 2x+5 < 3$$

$$-5 \quad -5 \quad -5$$

$$-12 \leq 2x < -2$$

$$\frac{-12}{2} \leq \frac{2x}{2} < \frac{-2}{2}$$

$$-6 \leq x < -1$$

Divide all 3 sides by 2:

$$\{x \mid -6 \leq x < -1\}$$

Ex. Solve. $1 < 1-3x < 19$

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$$0 < -3x < 18$$

$$\frac{0}{-3} > \frac{-3x}{-3} > \frac{18}{-3}$$

$$0 > x > -6$$

[Reverse both inequality signs!]

Rewrite so it's smallest to largest:

$$-6 < x < 0$$



$$\{x \mid -6 < x < 0\}$$

$$(-6, 0)$$

Ex. Solve.

$$-4 < 5(3-x) \leq 1$$

$$-4 < 15-5x \leq 1$$

$$-19 < -5x \leq -14$$

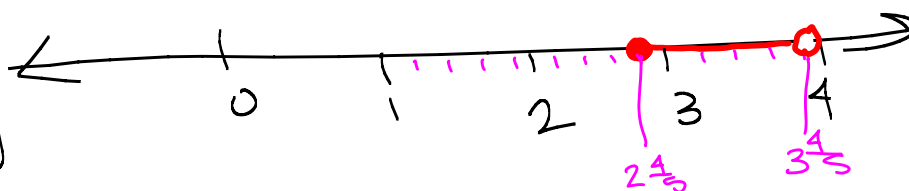
$$\frac{-19}{-5} > \frac{-5x}{-5} \geq \frac{-14}{-5}$$

$$\frac{19}{5} > x \geq \frac{14}{5}$$

$$\frac{14}{5} \leq x < \frac{19}{5}$$

$$2\frac{4}{5} \leq x < 3\frac{4}{5}$$

$$x \mid 2\frac{4}{5} \leq x < 3\frac{4}{5}$$



$$\left[\frac{14}{5}, \frac{19}{5}\right)$$

or

$$\left[2\frac{4}{5}, 3\frac{4}{5}\right)$$

Example: Solve

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$$-4 \leq \frac{2}{3}x + \frac{1}{2} \leq \frac{9}{2}$$

Multiply all three sides by 6 to clear the fractions:

$$-4(6) \leq \frac{2}{\cancel{3}^1}(\cancel{6}^2)x + \frac{1}{\cancel{2}^1}(\cancel{6}^2) \leq \frac{9}{\cancel{2}^1}(\cancel{6}^2)$$

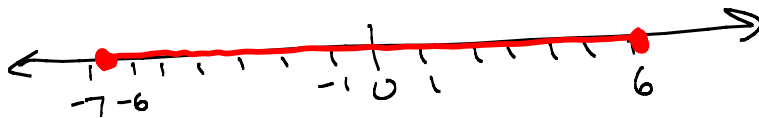
$$-24 \leq 4x + 3 \leq 27$$

$$-27 \leq 4x \leq 24$$

$$\frac{-27}{4} \leq \frac{4x}{4} \leq \frac{24}{4}$$

$$-6\frac{3}{4} \leq x \leq 6$$

$$\left\{ x \mid -6\frac{3}{4} \leq x \leq 6 \right\}$$



$$\left[-6\frac{3}{4}, 6 \right]$$