

5.3: Definitions of b^0 and b^{-n}

5.3.1

Note Title

10/3/2018

(Negative Exponents)

Definition of a negative exponent:

$$x^{-n} = \frac{1}{x^n}$$

Note: $x^{-n} = \frac{x^{-n}}{1} = \frac{1}{x^n}$

Also: $\frac{1}{x^{-n}} = \frac{x^n}{1}$

Simplify.

Example: $5^{-2} = \frac{5^{-2}}{1} = \frac{1}{5^2} = \boxed{\frac{1}{25}}$

In all the following examples, simplify. Write all answers without negative exponents.

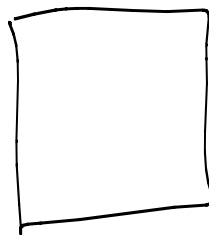
Ex: $-2^{-4} = -\frac{2^{-4}}{1} = -\frac{1}{2^4} = \boxed{-\frac{1}{16}}$

Ex: $(-3)^{-4} = \frac{(-3)^{-4}}{1} = \frac{1}{(-3)^4} = \boxed{\frac{1}{81}}$

Ex: $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$

OR $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$

Ex: $\frac{1}{4^{-2}} = \frac{4^2}{1} = \frac{16}{1} = \boxed{16}$



Ex. $2x^{-5} = \frac{2x^{-5}}{1} = \boxed{\frac{2}{x^5}}$

Ex. $(2x)^{-5} = \frac{(2x)^{-5}}{1} = \frac{2^{-5} x^{-5}}{1} = \frac{1}{2^5 x^5}$
 $= \boxed{\frac{1}{32x^5}}$

Ex. $a^{-5} a^3 = \frac{a^{-5} a^3}{1} = \frac{a^3}{a^5} = \boxed{\frac{1}{a^2}}$

Ex. $(a^3 b^{-4})^{-5}$
 $= \frac{(a^3 b^{-4})^{-5}}{1} = \frac{a^{-15} b^{20}}{1}$
 $= \boxed{\frac{b^{20}}{a^{15}}}$

Ex. $\frac{t^2}{t^{-8}} = \frac{t^2 t^8}{1} = \frac{t^{10}}{1} = \boxed{t^{10}}$

$\rightarrow$ or $\frac{(a^3 b^{-4})^{-5}}{1} = \frac{1}{(a^3 b^{-4})^5}$
 $= \frac{1}{a^{15} b^{-20}} = \boxed{\frac{b^{20}}{a^{15}}}$

Ex. Simplify 2^0 .

5.3.4

Notice:

$$2^5 = 32$$

$$2^4 = 32 \div 2 = 16$$

$$2^3 = 16 \div 2 = 8$$

$$2^2 = 8 \div 2 = 4$$

$$2^1 = 4 \div 2 = 2$$

$$2^0 = 2 \div 2 = \frac{2}{2} = 1$$

$$2^{-1} = 1 \div 2 = \frac{1}{2}$$

$$2^{-2} = \frac{1}{2} \div 2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Important Fact:

$x^0 = 1$, as long as $x \neq 0$.

Also note:

$$\frac{2^3}{2^3} = \frac{8}{8} = 1$$

From property of exponents,

$$\frac{2^3}{2^3} = 2^{3-3} = 2^0$$

must be equal!

5.3.5

Ex: $(-5)^0 = \boxed{1}$

Ex: $\left(\frac{3753}{8}\right)^0 = \boxed{1}$

Ex: $(4x^2y^3z)^0 = \boxed{1}$, as long as $x \neq 0$, $y \neq 0$, $z \neq 0$

Note: 0^0 is undefined.

Ex: Simplify.
 $\left(\frac{2x^{-4}}{3}\right)^{-4} = \frac{2^{-4} x^{16}}{3^{-4}} = \frac{3^4 x^{16}}{2^4} = \boxed{\frac{81x^{16}}{16}}$

Ex: $\frac{x^{-3}y^4z}{x^{-8}y^9z^{-2}} = \frac{y^4z x^8}{x^3 y^9} \cdot \frac{z^2}{z^2} = \frac{x^8 y^4 z z^2}{x^3 y^9}$
 $= \frac{x^8 y^4 z^3}{x^3 y^9} = \boxed{\frac{x^5 z^3}{y^5}}$

5.3.6

Ex.
$$\frac{-3x^{-5}(x^{-1}y^3z^{-2})^{-4}}{(3x^2y^4)^{-2}(2x^{-1}y^{-3}z)^4}$$

$$= \frac{-3x^{-5}x^4y^{-12}z^8}{3^{-2}x^{-4}y^{-8} \cdot 2^4x^4y^{-12}z^4}$$

$$= \frac{-3x^4z^8 \cdot 3^2x^4y^0}{\cancel{x^5}y^{12} \cdot \cancel{2^4}x^4\cancel{y^{12}}z^4}$$

$$= \frac{-3(9)x^8y^{20}z^8}{2^4x^9y^{12}z^4} = \boxed{\frac{-27y^8z^4}{16x}} \text{ or } \boxed{-\frac{27y^8z^4}{16x}}$$