5.6: Multiplying Polynomials (cont'd)

Example: multiply.

$$(4x-3)(-2x^2+y-x)$$

$$= 4x(-2x^2+y-x)-3(-2x^2+y-x)$$

$$= -8x^3+4xy-4x^2+6x^2-3y+3x$$

$$= \boxed{-8x^3+4xy+2x^2-3y+3x}$$

Ex: $(x^2-7x-1)(2x^2-5x-6)$

$$= x^2(2x^2-5x-6)-7x(2x^2-5x-6)-1(2x^2-5x-6)$$

$$= 2x^4-5x^3-6x^2$$
$$-14x^3+35x^2+42x$$
$$-2x^2+5x+6$$

$$= \boxed{2x^4-19x^3+27x^2+47x+6}$$

Ex: $(-3x^2+6x-4)(x^2-x-7)$

$$= -3x^2(x^2-x-7)+6x(x^2-x-7)-4(x^2-x-7)$$

$$= -3x^4+3x^3+21x^2$$
$$+6x^3-6x^2-42x$$
$$-4x^2+4x+28$$

$$= \boxed{-3x^4+9x^3+11x^2-38x+28}$$

5.6.4

The "FOIL" method

(for multiplying 2 binomials)

F : First
O : Outer
I : Inner
L : Last

Ex: $(2x+3)(3x-4)$

$= 6x^2 - 8x + 9x - 12$

$= \boxed{6x^2 + x - 12}$

Ex: $(x^2+6)(x-8)$

$= \boxed{x^3 - 8x^2 + 6x - 48}$

Difference of 2 Squares Pattern

Ex: $(x-5)(x+5)$

$= x^2 + \cancel{5x} - \cancel{5x} - 25$

$= \boxed{x^2 - 25}$

Difference of Two squares pattern:

$$(a+b)(a-b) = a^2 - b^2$$

Perfect squares

Ex: $(x-7)^2$

$= (x-7)(x-7)$

$= x(x-7) - 7(x-7)$

$= x^2 - 7x - 7x + 49$

$= \boxed{x^2 - 14x + 49}$

Note: $(x+y)^2 \neq x^2 + y^2$

$(xy)^2 = x^2 y^2$

5.7: Division of Polynomials

Dividing a polynomial by a monomial

Example: Divide.

$$\frac{6x^4 - 8x^3 + 12x^2 - 9}{2x^2}$$

Split into separate fractions:

$$\frac{6x^4 - 8x^3 + 12x^2 - 9}{2x^2} = \frac{\overset{3}{\cancel{6}}x^4}{\underset{1}{\cancel{2}}x^2} - \frac{\overset{4}{\cancel{8}}x^3}{\underset{1}{\cancel{2}}x^2} + \frac{\overset{6}{\cancel{12}}x^2}{\underset{1}{\cancel{2}}x^2} - \frac{9}{2x^2}$$

$$= \frac{3x^2}{1} - \frac{4x}{1} + \frac{6}{1} - \frac{9}{2x^2}$$

$$= \boxed{3x^2 - 4x + 6 - \frac{9}{2x^2}}$$

Note:

$$\frac{2}{7} + \frac{3}{7}$$

$$= \frac{2+3}{7}$$

$$= \frac{5}{7}$$

Ex:

$$\frac{2x^2 + 5xy + 18y^4}{4x^2y^3}$$

$$= \frac{\overset{1}{\cancel{2}}x^2}{\underset{2}{\cancel{4}}x^2y^3} + \frac{5xy}{4x^2y^3} + \frac{\overset{9}{\cancel{18}}y^4}{\underset{2}{\cancel{4}}x^2y^3}$$

$$= \boxed{\frac{1}{2y^3} + \frac{5}{4xy^2} + \frac{9y}{2x^2}}$$

Dividing a polynomial by a polynomial
(where divisor has 2 or more terms)

If divisor has 2 or more terms, we need

Polynomial Long Division

Review: Long Division with numbers

Example: Divide $\frac{379}{12}$. (equivalently, $379 \div 12$)

$$\begin{array}{r} 31 \\ 12\overline{)379} \\ -36 \\ \hline 19 \\ -12 \\ \hline 7 \end{array}$$

So, $\frac{379}{12} = \boxed{31 + \frac{7}{12}}$ (usually written $31\frac{7}{12}$)

Terminology review:

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor}\overline{)\text{Dividend}} \\ \hline \text{Remainder} \end{array}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$

Example: Divide.

$$\frac{3x^2+7x+9}{x+2}$$

Divisor has 2 terms, so I need long division

$$\frac{3x^2}{x} \qquad \frac{x}{x}$$

$$
\begin{array}{r l}
 & 3x + 1 \\
x+2 & \overline{)\,3x^2 + 7x + 9} \\
 & -(3x^2 + 6x) \\ \hline
 & \qquad x + 9 \\
 & \quad -(x + 2) \\ \hline
 & \qquad\quad 7
\end{array}
$$

$3x(x+2)$

$x(?) = 3x^2$

Answer: $3x$

$1(x+2)$

$x(?) = x$

answer: 1

Write answer:

$$\frac{3x^2+7x+9}{x+2} = \boxed{3x+1+\frac{7}{x+2}}$$

5.7.4

Example: $\dfrac{x^3 - 6x^2 + x + 14}{x - 5}$ Divide.

$$\frac{x^3}{x} \qquad \frac{-x^2}{x} \qquad \frac{-4x}{x}$$

$$
\begin{array}{r|llll}
 & x^2 & -x & -4 & \\
\hline
x-5 & x^3 & -6x^2 & +x & +14 \\
 & -(x^3 & -5x^2) & & \longleftarrow x^2(x-5) \\
\hline
 & & -x^2 & +x & \\
 & & -(-x^2 & +5x) & \longleftarrow -x(x-5) \\
\hline
 & & & -4x & +14 \\
 & & & -(-4x & +20) \\
\hline
 & & & & -6
\end{array}
$$

Write answer:

$$\frac{x^3 - 6x^2 + x + 14}{x - 5} = \boxed{x^2 - x - 4 + \frac{-6}{x-5}}$$

OR $\boxed{x^2 - x - 4 - \dfrac{6}{x-5}}$

Check your answer by multiplying:

$$(x-5)(x^2 - x - 4) - 6$$

$$= x^3 - x^2 - 4x - 5x^2 + 5x + 20 - 6 = x^3 - 6x^2 + x + 14$$

✓ok (matches original dividend)

Ex: Divide $\dfrac{8x^3-10x^2+x-2}{2x-1}$

$$\frac{8x^3}{2x} \quad \frac{-6x^2}{2x} \quad \frac{-2x}{2x}$$

$$\begin{array}{r|l}
 & 4x^2 \;-3x\;-1 \\
\hline
2x-1 & 8x^3 \;-10x^2 \;+x \;-2 \\
 & -(8x^3 \;-4x^2) \\
\hline
 & -6x^2 \;+x \\
 & -(-6x^2 \;+3x) \\
\hline
 & -2x \;-2 \\
 & -(-2x \;+1) \\
\hline
 & -3
\end{array}$$

$$\frac{8x^3-10x^2+x-2}{2x-1} = \boxed{4x^2-3x-1+\frac{-3}{2x-1}}$$

Check it: $(2x-1)(4x^2-3x-1)-3$

$= 8x^3-6x^2-2x$
$\quad -4x^2+3x+1-3$

$= 8x^3-10x^2+x-2$ ✓ok matches the numerator

Important: Arrange terms in descending order.

Insert "placeholders" (Ex $0x^2$, $0x^3$) for missing powers.

Ex: Divide: $\dfrac{3x^4 - 50x^2 - x}{x-4}$

$\frac{3x^4}{x}$ $\quad \frac{12x^3}{x}$ $\quad \frac{-2x^2}{x}$ $\quad \frac{-9x}{x}$

$$
\begin{array}{r|l}
 & 3x^3 + 12x^2 - 2x - 9 \\
x-4 & 3x^4 + 0x^3 - 50x^2 - x + 0 \\
 & -(3x^4 - 12x^3) \\
 & 12x^3 - 50x^2 \\
 & -(12x^3 - 48x^2) \\
 & -2x^2 - x \\
 & -(-2x^2 + 8x) \\
 & -9x + 0 \\
 & -(-9x + 36) \\
 & -36
\end{array}
$$

$3x^3(x-4) = 3x^4 - 12x^3$

$12x^2(x-4)$

$-2x(x-4)$

write answer:

$$\frac{3x^4 - 50x^2 - x}{x-4} = \boxed{3x^3 + 12x^2 - 2x - 9 + \frac{-36}{x-4}}$$

$$= \boxed{3x^3 + 12x^2 - 2x - 9 - \frac{36}{x-4}}$$

Check: $(x-4)(3x^3 + 12x^2 - 2x - 9) - 36$

$= 3x^4 + \cancel{12x^3} - 2x^2 - 9x$
$\quad \cancel{-12x^3} - 48x^2 + 8x + \cancel{36} - \cancel{36}$

$= 3x^4 - 50x^2 - x$ ✓ok