6.1: Greatest Common Factor and Factoring by Grouping

Note Title 10/22/2018

Expressions being added are called <u>terms</u>.

Expressions being multiplied are called <u>factors</u>.

In Chapter 6, we will break polynomials down into their factors.

Unfactored form: $x^2 + 7x + 12$

Factored form: $(x+3)(x+4)$

$x+3$ and $x+4$ are factors of $x^2 + 7x + 12$.

<u>Recall</u>: Prime factorization of numbers:

<u>Ex</u>: Find the prime factorization of 220.

<u>Prime number</u>: Its only factors are 1 and itself.

220

$11 \cdot 20$

$11 \cdot 2 \cdot 10$

$11 \cdot 2 \cdot 2 \cdot 5$

So $220 = \boxed{11 \cdot 2^2 \cdot 5}$

The Greatest Common Factor (GCF)

Ex: What is the GCF of 12 and 20?
(What's the largest number that divides into both of them?)

$\boxed{4}$

To find the GCF of a polynomial:
- Find the GCF of all the coefficients.
- Find the largest power of each variable that is a factor of all the terms.
  If a variable appears in all the terms, choose the smallest exponent.

Ex: Factor out the GCF of $2x^5 + 6x^3$.

GCF: $2x^3$

$$2x^5 + 6x^3$$
$$= 2x^3\left(\frac{\cancel{2}^{1}x^5}{\cancel{2}x^3} + \frac{\cancel{6}^{3}x^3}{\cancel{2}x^3}\right)$$
$$= \boxed{2x^3(x^2 + 3)}$$

Check: $2x^3(x^2 + 3)$
$= 2x^5 + 6x^3$ ✓

Ex: Factor $32x^7 + 48x^8y$

~~GCF~~ GCF: $8x^7$

GCF: $16x^7$

$$8x^7\left(\frac{32x^7}{8x^7} + \frac{48x^8y}{8x^7}\right)$$

$$= 8x^7(4 + 6xy)$$

Can still divide out 2: $= 8x^7(2)(2+3xy)$

$$= \boxed{16x^7(2+3xy)}$$

Ex: Factor $12x^2 + 4x + 18$.

GCF: 2

$$12x^2 + 4x + 18$$

$$= \boxed{2(6x^2 + 2x + 9)}$$

Check: $12x^2 + 4x + 18$ ✓

Ex: Factor.

$$8x^3y^4 + 12x^2y^3 + 20xy^2z - 24x^3y^2$$

$$= \boxed{4xy^2(2x^2y^2 + 3xy + 5z - 6x^2)}$$

Check: $8x^3y^4 + 12x^2y^3 + 20xy^2z - 24x^3y^2$

Ex: Factor $x^5 + x^3$.

$$x^5 + x^3$$
$$= x^3\left(\frac{x^5}{x^3} + \frac{x^3}{x^3}\right)$$
$$\boxed{= x^3(x^2+1)}$$

Check it!

Factoring by Grouping

If a polynomial has 4 terms, arrange it into two groups and factor out the GCF for each group.

Ex: Factor.

GCF: 1

$$2x^3 + 3x^2 + 10x + 15$$
$$= (2x^3 + 3x^2) + (10x + 15)$$
$$= x^2(2x+3) + 5(2x+3)$$
$$= \boxed{(2x+3)(x^2+5)}$$

Note: $x^2y + 5y$
$= y(x^2+5)$

Check by multiplying: $(2x+3)(x^2+5)$
$= 2x^3 + 10x + 3x^2 + 15$ ✓ equal to original

Also correct: $\boxed{(x^2+5)(2x+3)}$

Ex: Factor. $9x^3 - 21x^2 + 3x - 7$

$$(9x^3 - 21x^2) + (3x - 7)$$

$$= 3x^2(3x - 7) + 1(3x - 7)$$

$$= \boxed{(3x-7)(3x^2+1)}$$

Check it!

Ex: Factor.

$$ax - 4a - 3x + 12$$

$$= (ax - 4a) + (-3x + 12)$$

$$= a(x - 4) - 3(x - 4)$$

$$= \boxed{(x-4)(a-3)} \quad \text{or} \quad \boxed{(a-3)(x-4)}$$

Ex: Factor.

$$2x^4 - 12x^3 - 16x^2 + 96x \qquad \text{GCF: } 2x$$

$$= 2x\left(x^3 - 6x^2 - 8x + 48\right)$$

$$= 2x\left[\left(x^3 - 6x^2\right) + \left(-8x + 48\right)\right]$$

$$= 2x\left[x^2\underline{(x-6)} - 8\underline{(x-6)}\right]$$

$$= 2x\left[(x-6)(x^2-8)\right]$$

$$= \boxed{2x(x-6)(x^2-8)}$$

OR $\quad 2x^4 - 12x^3 - 16x^2 + 96x$

$$= \left(2x^4 - 12x^3\right) + \left(-16x^2 + 96x\right)$$

$$= 2x^3(x-6) - 8x(2x - 12)$$

$$= 2x^3(x-6) - 16x(x-6)$$

$$= (x-6)(2x^3 - 16x)$$

Can factor out $2x$ from $2x^3 - 16x$

$$= (x-6)(2x)(x^2-8)$$

$$= \boxed{2x(x-6)(x^2-8)}$$

6.2: Factoring Trinomials (Leading Coefficient is 1)

Ex: Factor.

(+) same signs (both pos or both neg)

GCF: 1

$$x^2 + 6x + 8$$

$$\boxed{(x+2)(x+4)}$$

8

1 · 8

2 · 4

Check: $x^2 + 4x + 2x + 8$

$= x^2 + 6x + 8$ ✓

(−) signs must be opposite

Ex: $x^2 + 10x - 24$

$$\boxed{(x-2)(x+12)}$$

24

1 · 24

2 · 12

3 · 8

4 · 6

Check: $(x-2)(x+12)$

$x^2 + 12x - 2x - 24$

$x^2 + 10x - 24$ ✓

Signs are opposite, so I want a difference of 10