From RR 21

Note Title 2/5/2015

1) $y = -\frac{2}{3}x + 2$

Recall: $y = mx + b$ (Slope-intercept form)

Slope: $m = -\frac{2}{3}$

y-intercept: $b = 2 \Rightarrow (0, 2)$

Slope: $m = -\frac{2}{3}$

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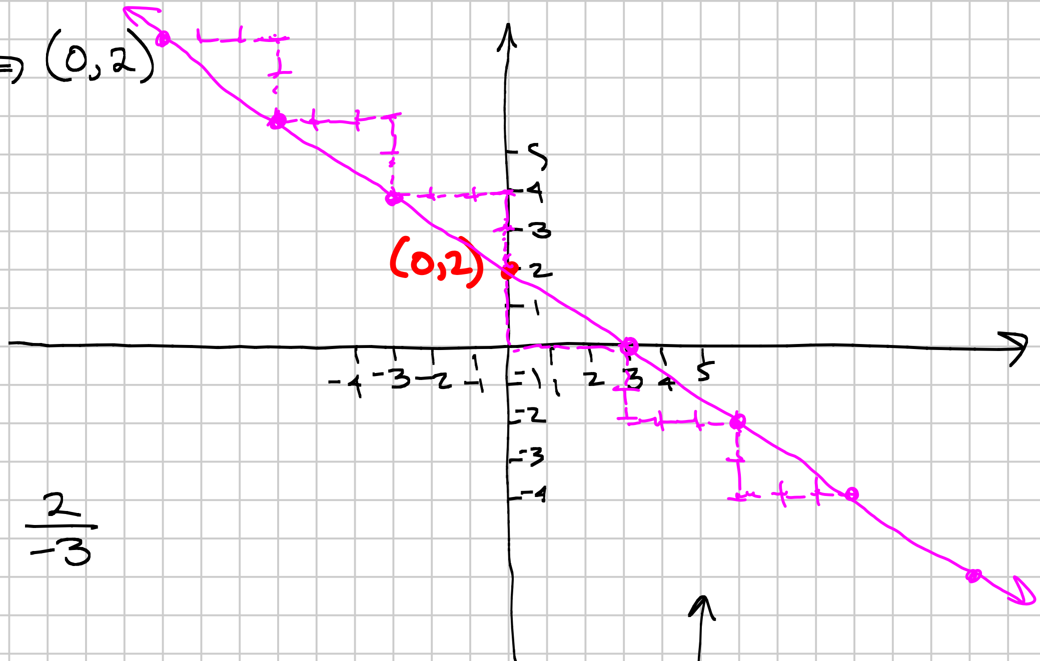

Rise: 2
Run: 3

Note: $-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$

2) $y = x - 1$ It's in $y = mx + b$ form

$y = 1x - 1$

Slope: $m = 1 = +\frac{1}{1}$

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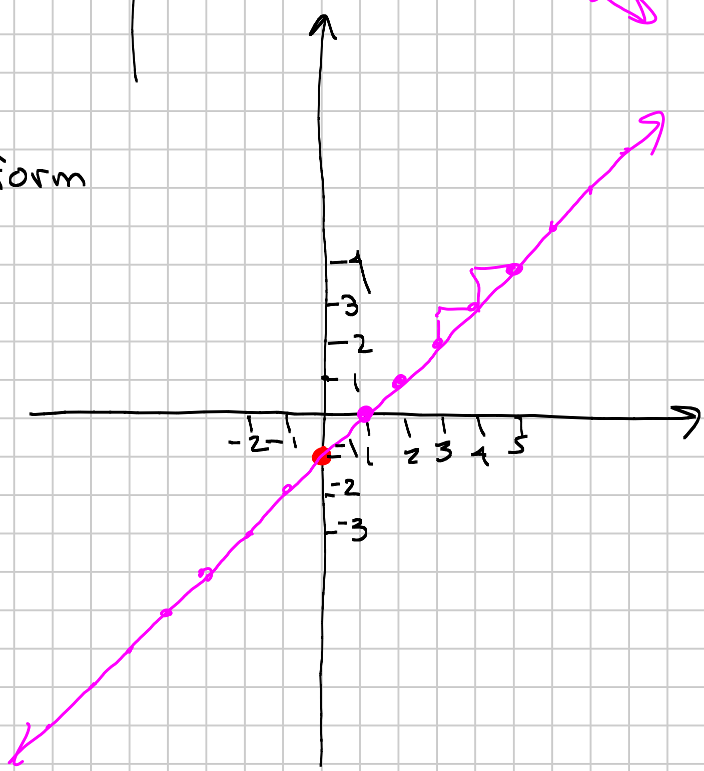


y-intercept: $b = -1 \Rightarrow (0, -1)$

3.4: The Slope-intercept form of a line (cont'd)

Ex. Graph the line $-5x + 3y = 9$ by writing in slope-intercept form.

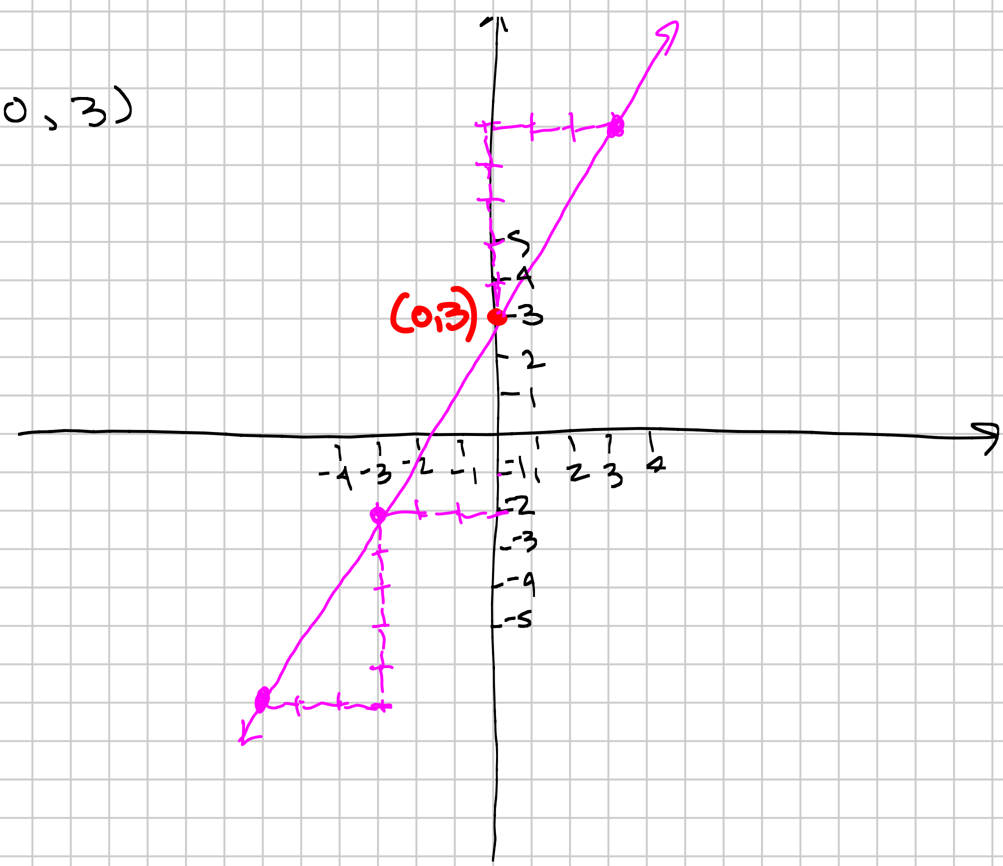
We want to rewrite as $y = mx + b$. Need to solve for y (isolate the y).

$$-5x + 3y = 9$$
$$+5x \qquad\qquad +5x$$
$$3y = 5x + 9$$
$$\frac{3y}{3} = \frac{5x}{3} + \frac{9}{3}$$
$$y = \frac{5}{3}x + 3$$

Slope: $m = +\frac{5}{3}$

$\nearrow +$

y-intercept: $b = 3 \Rightarrow (0, 3)$



4.1: Solving Linear Systems Using Graphing

4.1.1

System of Equations: A group of 2 or more equations.

Solution to a System (of equations): A set of values for the variables that make all the equations true.

For us: We will work with systems of 2 linear equations in 2 variables (typically x and y). (linear systems)

Example:
$$\begin{cases} 2x - 5y = 4 \\ x + 6y = 8 \end{cases}$$

Note: graphs of these equations are lines

Example: Is $(9, -2)$ a solution of this system?

$$\begin{cases} 2x + 5y = 8 \\ 3x - 2y = 23 \end{cases}$$

Put $x = 9$, $y = -2$ into $2x + 5y = 8$:

$$2(9) + 5(-2) = 8$$
$$18 - 10 = 8$$
$$8 = 8 \quad \checkmark \text{ True!}$$

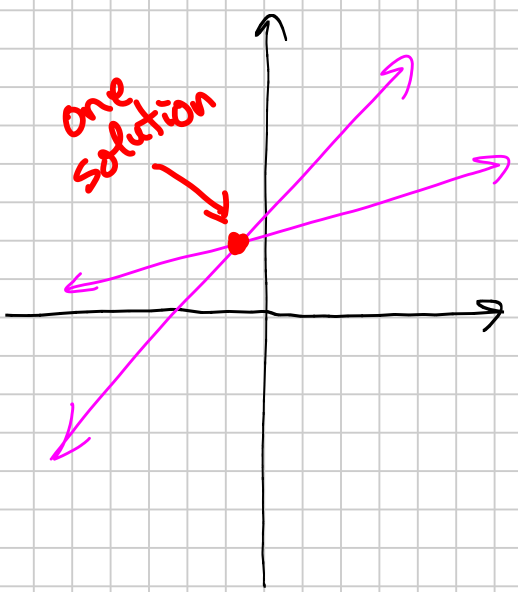
$(9, -2)$ is a solution to the 1st eqn.

Put $x = 9$, $y = -2$ into $3x - 2y = 23$:

$$3(9) - 2(-2) = 23$$
$$27 + 4 = 23$$
$$31 = 23 \quad \text{False!}$$

So $\boxed{\text{No, } (9,-2) \text{ is not a solution to the system.}}$

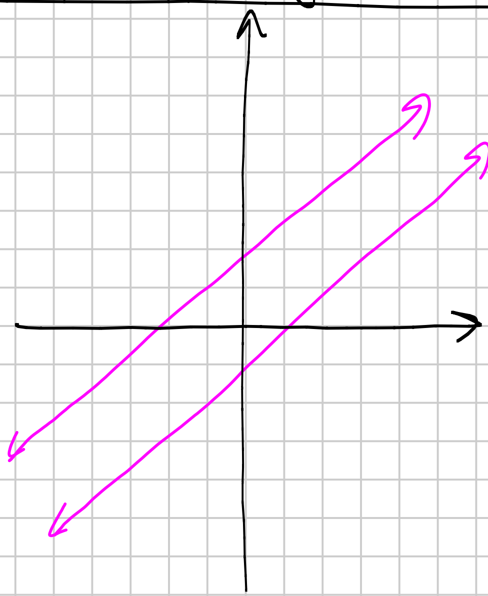
3 possible situations for a system of 2 linear equations



Independent System

one Solution

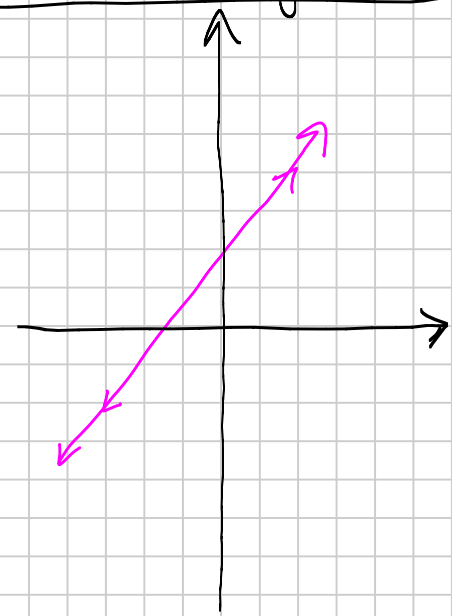
(lines intersect at a single point)



Inconsistent System

No Solution

(lines are parallel)



Dependent System

Infinitely Many solutions

(lines are the same)

Question on Test #4:

For an independent system, an inconsistent system, and/or a dependent system:

1) State the number of solutions.
2) Describe the graph in words (lines cross at 1 point, lines are parallel, lines are the same)
3) Illustrate with an example graph.

To solve a system by graphing, graph both lines and then estimate the solution from the graph.

Ex: Solve the system by graphing.

$$\begin{cases} x + 2y = 8 \\ x - 3y = 3 \end{cases}$$

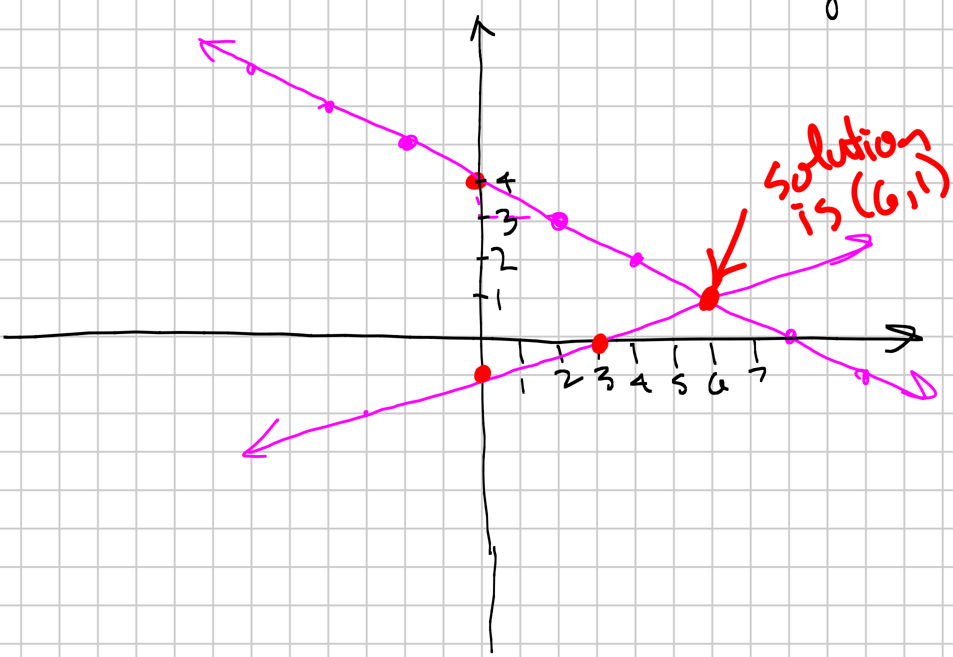
1st graph the line $x + 2y = 8$

Write as $y = mx + b$:

$$x + 2y = 8$$
$$\quad -x \qquad -x$$
$$2y = -x + 8$$
$$\frac{2y}{2} = \frac{-x}{2} + \frac{8}{2}$$
$$y = -\frac{1}{2}x + 4$$

Slope: $m = -\frac{1}{2}$

y-intercept: $b = 4 \Rightarrow (0, 4)$



Next graph the line $x - 3y = 3$:

Could write this one as $y = mx + b$ also.

or: Review: graph using intercepts:

Find the y-intercept:

Set $x = 0$:

$$x - 3y = 3$$
$$0 - 3y = 3$$
$$-3y = 3$$
$$\frac{-3y}{-3} = \frac{3}{-3}$$
$$y = -1 \Rightarrow (0, -1)$$

Previous example cont'd:

Find x-intercept:

Set $y=0$: $x-3y=3$

$x-3(0)=3$

$x-0=3$

$x=3 \implies (3,0)$

From graph, solution appears to be $(6,1)$.

Check it:

$x=6,\ y=1 \implies$

$x+2y=8$

$6+2(1)=8$

$6+2=8$

$8=8$ ✓

$x=6,\ y=1 \implies$

$x-3y=3$

$6-3(1)=3$

$6-3=3$

$3=3$ ✓

$\boxed{\text{The Solution is } (6,1).}$

This is an independent system.

4.2: Solving Linear Systems with the Substitution Method

Solving by substitution (step-by-step):

1) Solve one equation (your choice) for one variable (your choice).
2) Substitute this expression into the other equation.
3) Solve for the remaining variable.
4) Plug this value into either equation and solve.
5) Check your answer.

Ex: Solve the system by substitution. 4.2.2

$$\begin{cases} x - y = 9 \\ 2x - 11y = -18 \end{cases}$$

1) Solve $x - y = 9$ for x:

$$\begin{aligned} x - y &= 9 \\ +y \quad &\quad +y \\ x &= y + 9 \end{aligned}$$

2) Substitute $x = y + 9$ into $2x - 11y = -18$:

$$2(\qquad) - 11y = -18$$

$$2(y + 9) - 11y = -18$$

3) Solve for y:

$$\begin{aligned} 2y + 18 - 11y &= -18 \\ -9y + 18 &= -18 \\ -18 \quad &\quad -18 \\ -9y &= -36 \\ \frac{-9y}{-9} &= \frac{-36}{-9} \\ y &= 4 \end{aligned}$$

4) Plug $y = 4$ into $x - y = 9$ and solve:

$$\begin{aligned} x - 4 &= 9 \\ +4 \quad &\quad +4 \\ x &= 13 \end{aligned}$$

$\Longrightarrow$ $\boxed{\text{Solution is } (13, 4).}$

5) Check it:

$x - y = 9$

$x = 13, y = 4 \Longrightarrow 13 - 4 = 9$

$9 = 9$ ✓ True!

$2x - 11y = -18$

$x = 13, y = 4 \Longrightarrow 2(13) - 11(4) = -18$

$26 - 44 = -18$

$\Longrightarrow -18 = -18$ ✓ True!

Ex: Solve by Substitution

4.2.3

$$\begin{cases} -4x + 2y = -4 \\ 2x + y = 8 \end{cases}$$

1) Solve $2x + y = 8$ for y:

$$2x + y = 8$$
$$-2x \qquad -2x$$
$$y = -2x + 8$$

2) Substitute $y = -2x + 8$ into $-4x + 2y = -4$:

$$-4x + 2(\qquad) = -4$$
$$-4x + 2(-2x + 8) = -4$$

3) Solve for x:

$$-4x - 4x + 16 = -4$$
$$-8x + 16 = -4$$
$$-16 \qquad -16$$
$$-8x = -20$$
$$\frac{-8x}{-8} = \frac{-20}{-8}$$
$$x = \frac{20}{8} = \frac{10}{4} = \frac{5}{2}$$

4) Plug $x = \frac{5}{2}$ into $2x + y = 8$:

$$\frac{2}{1}\left(\frac{5}{2}\right) + y = 8$$
$$\frac{10}{2} + y = 8$$
$$5 + y = 8$$
$$-5 \qquad -5$$
$$y = 3 \implies \boxed{\text{Solution is } \left(\frac{5}{2}, 3\right).}$$

5) Check: $-4x + 2y = -4$

$$-\frac{4}{1}\left(\frac{5}{2}\right) + 2(3) = -4$$
$$-\frac{20}{2} + 6 = -4$$
$$-4 = -4 \Leftarrow -10 + 6 = -4 \checkmark \text{ True!}$$

$2x + y = 8$

$$2\left(\frac{5}{2}\right) + 3 = 8$$
$$5 + 3 = 8$$
$$8 = 8 \checkmark$$