From RR22

Note Title 11/28/2018

Solve by substitution:

$$\begin{cases} 5x - 3y = -2 \\ 10x - y = 1 \end{cases}$$

Solve $10x - y = 1$ for y:

$$\begin{aligned} 10x - y &= 1 \\ -10x \quad &\quad -10x \end{aligned}$$

$$-y = 1 - 10x$$

$$\frac{-y}{-1} = \frac{1}{-1} - \frac{10x}{-1}$$

$$y = -1 + 10x \quad \text{or} \quad y = 10x - 1$$

Substitute $y = -1 + 10x$ into $5x - 3y = -2$.

$$5x - 3(-1 + 10x) = -2$$

$$5x + 3 - 30x = -2$$

$$-25x + 3 = -2$$

$$-3 \quad -3$$

$$-25x = -5$$

$$\frac{-25x}{-25} = \frac{-5}{-25}$$

$$x = \frac{1}{5}$$

See next page

RR 22 Problem cont'd

Put $x = \frac{1}{5}$ into either equation.

Put $x = \frac{1}{5}$ into $10x - y = 1$.

$$\cancel{10}^{2}\left(\frac{1}{\cancel{5}}\right) - y = 1$$

$$2 - y = 1$$

$$-2 \qquad -2$$

$$-y = -1$$

$$\frac{-y}{-1} = \frac{-1}{-1}$$

$$y = 1$$

$$\boxed{\text{Solution is } \left(\tfrac{1}{5}, 1\right).}$$

Check it:

$$5x - 3y = -2$$

$$\cancel{5}\left(\frac{1}{\cancel{5}}\right) - 3(1) = -2$$

$$1 - 3 = -2$$

$$-2 = -2 \quad \checkmark$$

$$10x - y = 1$$

$$x = \frac{1}{5},\ y = 1 \Rightarrow \cancel{10}^{2}\left(\frac{1}{\cancel{5}}\right) - 1 = 1$$

$$2 - 1 = 1$$

$$1 = 1 \quad \checkmark$$

4.3: Solving systems by the elimination/addition method:

Steps for solving by elimination/addition:

1) Choose a variable to eliminate. Then multiply one or both equations by a strategic number, so that when you add the equations, the variable will disappear.
   (Look for the Least Common Multiple of the coefficients)

2) Add the equations. One variable should disappear.

3) Solve for the remaining variable.

4) Either (a) Repeat Steps 1-3 for the other variable,
   or
   (b) Plug your value into either equation and solve.

5) Check your solution.

Ex: Solve the system using the elimination method (addition method).

$$\begin{cases} 11x - 5y = 2 \\ 3x - y = 1 \end{cases}$$

$$\begin{aligned} 11x - 5y &= 2 &\longrightarrow\quad 11x - 5y &= 2 \\ 3x - y &= 1 &\xrightarrow{(-5)}\quad -15x + 5y &= -5 \end{aligned}$$

Add: $-4x + 0 = -3$

$$-4x = -3$$

$$\frac{-4x}{-4} = \frac{-3}{-4}$$

$$x = \frac{3}{4}$$

Could plug $x = \frac{3}{4}$ into either equation, or do elimination again to get 2nd variable:

$$\begin{aligned} 11x - 5y &= 2 &\xrightarrow{(-3)}\quad -33x + 15y &= -6 \\ 3x - y &= 1 &\xrightarrow{(11)}\quad 33x - 11y &= 11 \end{aligned}$$

Add: $0 + 4y = 5$

$$4y = 5$$

$$\frac{4y}{4} = \frac{5}{4}$$

$$y = \frac{5}{4} \implies \boxed{\text{Solution is } \left(\frac{3}{4}, \frac{5}{4}\right)}$$

See next page for check

Previous example cont'd:

Check your answer:

$$11x - 5y = 2$$

$$x = \frac{3}{4},\ y = \frac{5}{4} \Rightarrow 11\left(\frac{3}{4}\right) - 5\left(\frac{5}{4}\right) = 2$$

$$\frac{33}{4} - \frac{25}{4} = 2$$

$$\frac{8}{4} = 2$$

$$2 = 2 \checkmark$$

$$3x - y = 1$$

$$x = \frac{3}{4},\ y = \frac{5}{4} \Rightarrow 3\left(\frac{3}{4}\right) - \frac{5}{4} = 1$$

$$\frac{9}{4} - \frac{5}{4} = 1$$

$$\frac{4}{4} = 1$$

$$1 = 1 \checkmark$$

Ex:

Solve the system by elimination.

4.3.4

$$\begin{cases} -5x + 2y = -6 \\ 7y + 10x = 34 \end{cases}$$

(done in class)

See previous semesters' notes.

4.3.5

Ex: Solve the system:

$$\begin{cases} -x+6y=2 \xrightarrow{(3)} -3x+18y=6 \\ 3x-18y=5 \longrightarrow 3x-18y=5 \end{cases}$$

Add:

$$0+0=11$$

$$0=11$$

False!

False statement, so $\boxed{\text{No Solution}}$

(lines are parallel)

$\boxed{\text{Inconsistent System}}$

Ex: Solve.

$$2x-10y=6 \xrightarrow{(5)} 10x-50y=30$$

$$-5x+25y=-15 \xrightarrow{(2)} -10x+50y=-30$$

$$0+0=0$$

$$0=0$$

True!

True statement, so this is a $\boxed{\text{dependent system}}$ (Lines are the same).

$\boxed{\text{Infinitely many solutions}}$

4.3.6

Summary:

1) If both variables disappear and you get a false statement (such as $0 = 11$ or $5 = -6$), the system is inconsistent and has no solution (lines are parallel).

2) If both variables disappear and you get a true statement (such as $0 = 0$ or $6 = 6$), the system is dependent and has infinitely many solutions (lines are the same).

3) If you get an ordered pair for a solution, then the system is independent. (lines intersect at one point)