2.2: Solving Linear Equations (cont'd) | 2.2.4

Note Title 9/12/2018

Example: Solve.

$$4x - \frac{3}{5} = \frac{1}{3}x + \frac{1}{2}$$

$$4x - \frac{1}{3}x = \frac{3}{5} + \frac{1}{2}$$

$$\frac{(3)}{(3)} \cdot \frac{4x}{1} - \frac{1}{3}x = \frac{3}{5} \cdot \frac{(2)}{(2)} + \frac{1}{2}\frac{(5)}{(5)}$$

$$\frac{12x}{3} - \frac{1x}{3} = \frac{6}{10} + \frac{5}{10}$$

$$\frac{11x}{3} = \frac{11}{10}$$

$$\frac{\frac{11}{3}x}{\frac{11}{3}} = \frac{\frac{11}{10}}{\frac{11}{3}}$$

$$\frac{3}{11}\left(\frac{11}{3}x\right) = \frac{11}{10}\left(\frac{3}{11}\right)$$

$$x = \frac{3}{10}$$

Conditional eqn

$\left\{\frac{3}{10}\right\}$

EX: Solve $3 - 8x = 5 - 8x$

$3 = 5 - 8x + 8x$

$3 = 5$ false

Contradiction

No solution

there is no value of x that makes the equation true

EX: Solve $4(2+x) = 2(2x+1) + 6$

$8 + 4x = 4x + 2 + 6$

$8 + 4x = 4x + 8$

$-4x \quad -4x$

$8 = 8$ True

always true no matter what x is

Sol'n set: All real numbers

Identity

EX:

$8 = 8$

$-8 \quad -8$

$0 = 0$ True

2.2.5

Ex: $-3(5+x) = 3(x-4) - 3$

$-15 - 3x = 3x - 12 - 3$

$-15 - 3x = 3x - 15$

$-3x \quad -3x$

$-15 - 6x = -15$

$+15 \quad +15$

$-6x = 0$

$\frac{-6x}{-6} = \frac{0}{-6}$

$x = 0$

Conditional equation

$\{0\}$

3 types of equations:

* Conditional Equation: eqn (equation) is true for one or more values of the variable, and false for all other values. (most common situation)
  You can isolate the variable and get $x =$ ____ / $1x =$ ____

* A contradiction: An eqn that is never true.
  Sol'n set: no solution.
  Variables disappear and leave you with a false statement: Ex: $6 = 10$ or $0 = 4$

* An identity: an eqn that is true for all values of the variable. Sol'n set: All real numbers
  Variables disappear & leave you with a true statement: Ex: $10 = 10$ or $0 = 0$

2.3: Clearing Fractions and Decimals

(An alternative approach to handling fractions)

To clear out the fractions, multiply both sides of the eqn by the least common denominator (LCD) of all the fractions.

(So multiply by the least common multiple of all the denominators)

Example: $7 = z - \frac{1}{2}$

$2(7) = 2(z - \frac{1}{2})$ [multiply both sides by 2]

$14 = 2z + 2(-\frac{1}{2})$

$14 = 2z - 1$

$+1 \qquad +1$

$15 = 2z$

$\frac{15}{2} = \frac{2z}{2}$

$z = \frac{15}{2}$

$\left\{\frac{15}{2}\right\}$

EX: Solve

$$\frac{1}{5}y + 3 = \frac{7}{10}$$

LCD: 10

$$10\left(\frac{1}{5}y\right) + 10(3) = \frac{7}{10}(10)$$

$$\frac{10}{5}y + 30 = \frac{70}{6}$$

$$2y + 30 = 7$$

$$2y = 7 - 30$$

$$2y = -23$$

$$\frac{2y}{2} = \frac{-23}{2}$$

$$y = -\frac{23}{2}$$

$$\left\{-\frac{23}{2}\right\}$$

EX: $\frac{3}{4}x - \frac{2}{3} + \frac{1}{6} = 4x - \frac{5}{2}$

Solve.

[multiply both sides by 12]

$$12\left(\frac{3}{4}x\right) - 12\left(\frac{2}{3}\right) + 12\left(\frac{1}{6}\right) = 12(4x) - 12\left(\frac{5}{2}\right)$$

$$9x - 8 + 2 = 48x - 30$$

$$9x - 6 = 48x - 30$$

$$-6 + 30 = 48x - 9x$$

$$24 = 39x$$

$$\frac{24}{39} = \frac{39x}{39}$$

$$\frac{8}{13} = x$$

$$x = \frac{8}{13}$$

$$\left\{\frac{8}{13}\right\}$$

Tip: If the sum of the digits is divisible by 3 (or 9), then the number is divisible by 3 (or 9).

To clear decimals, multiply both sides by a power of 10.

Ex: Solve. $3(-0.8x + 0.2) = -3.6x + 1.3$

$-2.4x + 1.2 = -3.6x + 1.3$

multiply both sides by 10:

$10(-2.4x) + 12(10) = -3.6x(10) + 1.3(10)$

$-24x + 12 = -36x + 13$

$+36x \quad -12 \qquad +36x \quad -12$

$12x = 1$

$\frac{12x}{12} = \frac{1}{12}$

$x = \frac{1}{12}$

$\left\{\frac{1}{12}\right\}$

Ex: $0.75(m-2) + 0.25m = 0.5$

multiply both sides by 100:

$100(0.75)(m-2) + 100(0.25m) = 0.50(100)$

$75(m-2) + 25m = 50$

$75m - 150 + 25m = 50$

$100m - 150 = 50$

$+150 \quad +150$

$100m = 200$

$m = \frac{200}{100} = 2$

Sol'n set: $\{2\}$